Does Extreme Correlation Matter in Global Equity Asset Allocation?

Bruno Solnik[1] and Thaisiri Watewai[2]

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Abstract

Global asset allocation provides risk diversification. But international market correlation increases sharply during global crises and diversification benefit disappears when it is most needed. We model these correlation breaks and derive the asset allocation implications. The model can quickly detect crises and suggests adapting allocation for changing correlation and volatility, as the crisis probability evolves. The out-of-sample results for ten major equity markets over 2008-2016 show significant improvements in the Sharpe ratio and maximum drawdown over mean-variance, fat-tail distribution, regime-switching, and 1/*N* rule. A benefit of the model is that it is conceptually intuitive and amenable to simple implementation in asset allocation and risk management.

Keywords: Extreme correlation, correlation break, global equity asset allocation, financial crises, regime switching

[1] Bruno Solnik is professor at the Finance Department, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. E-mail: solnik@ust.hk.
[2] Thaisiri Watewai is assistant professor at the Department of Banking and Finance, Chulalongkorn Business School, Chulalongkorn University, Bangkok, Thailand 10330. E-mail: thaisiri@cbs.chula.ac.th.

Since Harry Markowitz (1952), practitioners have based their optimization of global equity allocation on Mean-Variance (MV) analysis or variants thereof. Undoubtedly, formulating expected returns (risk premia) is a crucial part of a successful asset allocation. Returns are hard to predict and risk should be managed. However, the risk side of optimizers has remained quite simple and static. Risk diversification has been a major motivation for global equity allocation. But the past decades have shown that equity markets go through prolonged periods of global crisis where returns are low while volatility and international correlation are high. With high correlation, the benefits of international diversification nearly disappear. To simplify, markets globally go through periods of good and bad regimes with very different volatility and correlation characteristics. The risk level and the benefits from international diversification vary markedly.

Past crises have taught us that breaks in correlation could lead to huge losses in global portfolios, as illustrated by LTCM in the 1998 crisis or the 2008-2009 crisis. Portfolios that appeared well diversified exhibited huge drops in value because most of their investments became highly correlated. To illustrate why correlation is a key driver of risk, let's consider a portfolio spread evenly between ten **uncorrelated investments**, each with a **volatility of 10%**. Then the **portfolio volatility is 3.16%.** If markets become more volatile with a **volatility doubling** from 10% **to 20%,** the **portfolio volatility goes up to 6.32%** as long as the correlation remains null. **But if correlation breaks and shoots up to one**, then the **portfolio volatility goes up to 20%**. The increase from 6.32% to 20% is solely explained by the correlation break. One of the stylized facts of global equity markets is that returns tend to be more correlated when volatilities are high or when the markets go down. Extreme correlation, i.e. correlation for large absolute returns, tends to be much higher for down-markets than for up-markets, as shown by Longin and Solnik (2001) and extensive subsequent research.[3]

Unfortunately, we are never sure in which regime the market will be in the next few months. Asset allocation optimization and risk management should reflect the time-variation in return distributions. Time-variation affects both strategic and tactical allocation. Simply basing the strategic allocation on some long-term risk measures does not do the trick. One must reflect the risk of crises as we do here. The long-term strategic allocation should take into account the risk of a crisis appearing in the future with time-varying risk parameters, in the spirit of Merton (1973) dynamic modeling of uncertain shifts in investment opportunities. The tactical asset allocation should reflect not only changes in expected returns but also the probability of a change in risk parameters.

Recent literature tries to propose practical asset allocation strategies that protect portfolios against financial crises. For example, Xiong, and Idzorek (2011) propose to use Conditional Value at Risk (CVaR), instead of variance, as it takes skewness and kurtosis into account, and show that M-CVaR improves the portfolio performance over MV during the 2008 crisis. However, as pointed out by Greiner (2012), a more interesting question, which is still left unanswered, is whether the extreme correlation or fat tail is the main contributor of the improvement.[4] Wang, Sullivan and Ge (2012) propose to time the market by reducing/increasing the risk limit in M-CVaR when a CVaR forecast is high/low. But market timing has not been

[3] As stressed by Forbes and Rigobon (2002), there can also be a spurious increase in correlation due to increase in volatility if correlation and volatility are not modelled carefully, as we do in our model.

[4] See also Xiong and Idzorek (2012) for a comment on this issue.

linked to shifts in extreme correlations. Kritzman, Page, and Turkington (2012) propose another market timing strategy by tilting the allocation weights based on regime shifts of market turbulence, inflation, and economic growth variables. Their suggestion on the direction of the tilts relies mostly on risk premium (i.e. return) rather than volatility or correlation. It seems the increased correlation during bad markets is somewhat left out in these strategies despite being a well-established empirical fact.

Deriving simple allocation rules from complex dynamic models with time-varying risk parameters has been proven technically impossible. In this paper we discuss an asset allocation model based on Solnik and Watewai (2016) that is tractable and provides interesting applications to global equity portfolios. Modelling extreme correlation, i.e. the level of correlation when market returns take extreme value (positive or negative), yields a significant improvement in global equity portfolios. While time-variation in volatility has been extensively studied, research on time-variation in international correlation is less developed.[5]

After a brief description of the model and estimation results, we derive optimal portfolio weights and present some out-of-sample tests against other simpler models, including the famous "1/*N*" rule. The analyses are based on the 10 largest equity markets over the period 2001-2016. The results show that our model is able to quickly detect starts of crises, and suggests better leverage position and allocation among the markets. As a result, it has much lower risk and higher return out-of-sample compared to the other strategies. We conclude by looking at practical implications for global asset allocation.

The Model

The dynamic model of Solnik and Watewai (2016) assumes that country returns are driven by a Gaussian normal component (diffusion process in continuous time) and random global shocks (Poisson jump process). There are two different regimes (Markov regime-switching process) and the regimes are unobservable but we can estimate the probability of being in a given regime. The parameters of the return distributions are different in each regime. In the empirical part we find that regimes are persistent and one regime (the "*bad*" regime) has much lower mean return and much higher volatility and correlation than the other regime (the "*good*" regime). This allows modeling return asymmetries with a focus on correlation.

One could object that each economic/financial crisis is somewhat different from the previous ones and is triggered by different and somewhat-unpredictable events. But financial price behaviour tends to be fairly similar under crisis: Lower risk tolerance, liquidity crisis affecting prices, higher volatility (typically 2 to 3 times), higher correlation across all asset classes (correlation break), and very rapid shift from good to bad regime.

In mathematical terms, the multivariate log-return process of assets 1 through n is denoted $\{R_t, t = 1,2, \dots\}$, where:

$$R_t = Z_t + \sum_{m=1}^{\Delta \mathrm{N_t}} \delta_t^m \qquad t = 1,2, \dots$$

[5] For a review of the literature, see Solnik and Watewai (2016).

The first return component $Z_t = [Z_{1,t}, \ldots, Z_{n,t}]'$ is an n–dimensional Gaussian random vector representing the diffusion component. The second component is the jump component where ΔN_t is the random number of jumps that occur during time period t for which the distribution follows a Poisson distribution and $\delta_t^m = [\delta_{1,t}^m, \ldots, \delta_{n,t}^m]'$ is the vector of random jump sizes from the m^{th} jump for which the distribution is *Gaussian*. The jump sizes δ_t^m are assumed to be independent across m. The distributions of the diffusion, the number of jumps, and the jump sizes depend on the market regime Y_t. For example, jumps may have more negative mean and higher variance during the bad regime than the good regime. Jumps are assumed to be systemic or global-wide. That is an arrival of a jump affects returns of all assets at the instant of the arrival, but the jump size of each asset can be different.[6]

Note that correlations between asset returns come from two components: the vector of correlated Gaussian random variables Z and the systemic jump with vector of correlated jump sizes δ. The correlations due to Z can be viewed as a measure of the degree of small comovements between asset returns. On the other hand, the correlations due to δ can be viewed as a measure of the degree of large comovements between asset returns. This enables us to explicitly investigate the regime shifts of different types of correlations.

Our model can be viewed as a generalized discrete-time version of Das and Uppal (2004) who consider these types of Gaussian systemic jumps. They, however, assume that the jump sizes δ_i are perfectly correlated and consider a single regime. Ang and Bekaert (2002) consider a regime-switching model but without jumps.

One of Solnik and Watewai (2016) contributions is to derive an efficient estimation method based on the framework of the expectation maximization (EM) algorithm.[7] Although tractable EM algorithms for certain regime switching models have been proposed in the literature, deriving a tractable EM algorithm for multivariate models with regime-switching and jumps is nontrivial. Nevertheless, they are able to obtain a tractable algorithm for a large number of assets.

Empirical Results

Data

We use MSCI weekly total return data in USD for the ten largest investable markets: Australia (AU), Canada (CA), France (FR), Germany (GE), Hong Kong (HK), Japan (JP), Spain (SP), Switzerland (SW), the United Kingdom (UK), and the United States (US). Our sample covers well over 90% of the market capitalization of the MSCI World index. China is a closed market but many Chinese companies are listed on the Hong Kong exchange. Weekly data is used to alleviate the nonsynchronicity problem (time difference), while monthly data would make it difficult to quickly detect changes in regime. The period is January 2001 to December 2016 (835 observations).

[6] It is technically easy to augment the model to include additional types of jumps such as country-specific or region-specific. However we find that adding those do not improve the empirical fit of the model.

[7] The EM algorithm is an iterative method for computing maximum likelihood estimators of model parameters when some variables are missing or unobservable. The algorithm has two steps, which are performed alternately until convergence. Further technical details can be found in Solnik and Watewai (2016).

Table 1 shows the summary statistics of weekly returns of each index in % per week. All index returns have negative skewness, confirming the asymmetric distributions of returns. The minimum and maximum values show that negative shocks could be much larger than positive shocks. Over the period, the most severe minimum weekly return is –34.30%, while the largest positive weekly return is +17.76%. Such extreme returns, say beyond three standard deviations, have an extremely small probability of occurring under the normal distribution (0.27%), but we do observe them repeatedly during crises. Jumps can help explain those extreme returns. Excess kurtosis (fat tails) is very large and ranges from 2.21 for Japan to 13.35 for Australia. Such high kurtosis could be explained by jumps and nonstationarity. Regime switching and jumps can address nonstationarity in return distributions.

Estimation Results

We don't report detailed parameter estimates for each country, but simply their averages averaged across the ten countries.[8] We also performed all kind of statistical tests not reported here.[9] Table 2 shows the average means, standard deviations and correlations of the return, diffusion and jump size processes. As we estimate the parameters of the diffusion and jump processes in the two regimes, we can look at the relative contribution of each process to equity returns in each regime. Relative to the good regime, the bad regime is characterized by lower return (-0.31% per week vs +0.26%), higher volatility (4.47% per week vs 2.35%) and higher correlation (0.76 vs 0.63). The big difference between the two regimes in terms of means, volatility and correlation is induced by jumps.

A crucial result is that regimes are persistent. Markets go through prolonged periods of good and bad regimes. Figure 1 shows the filtered (dashed line) and smoothed (solid line) probabilities of being in the bad regime. The probability to stay in the same regime is high (above 95%), and the expected regime duration is long, as shown in Table 3. The expected duration is 25 weeks for the bad regime and 66 weeks for the good regime. Detecting a regime shift at an early stage enables the investors to adjust their portfolios for the new lengthy regime in which international correlation and volatility, among others, change drastically. A large negative jump can lead to a rapid transition to the bad regime, and the jump regime is persistent, so a crisis can be quickly detected. The benefit of early detection is confirmed by the out-of-sample results below.

We can gain some insights on regimes by looking at Figure 2A which charts periods of bad regime (grey) and the returns on an equal-weight global portfolio. The model quickly detects the start of a bad regime. This is true in all bad periods detected in the model. The increased correlation and volatility due to persistent big jumps allow for an early detection of bad regimes. A bull market is characterized by much smoother returns, and smoothness takes time to be confirmed. Furthermore, in the early stage of recovery, returns keep being quite correlated with

[8] Detailed results are available from the authors.

[9] We tested whether a regime-switching model with jumps is a significant improvement over the models used in the past literature: (1) one regime without jumps (mean-variance), (2) one regime with jumps, (3) two regimes without jumps, and (4) two regimes with jumps (our model). We strongly reject simpler models. We tested if a third global regime was significant but found that it was not. We added country or regional jumps but failed to reject our model with only systemic jumps. Hence we opted for the more parsimonious specification with two regimes and systemic jumps.

fairly large volatility. Hence our model fails to quickly detect return to a bull market. On the other hand, large negative systemic shocks easily signal a bad regime.

Changes in volatility help detect regimes but international correlation improves detection. This is also illustrated on Figure 2B. There are few high-frequency indicators of financial crisis, and they are domestic ones, primarily from the United States. We focus on the STLFSI (Saint Louis Fed's Financial Stress Index), which is a broad weekly index of financial stress, derived from a principal components analysis of eighteen financial indicators based on market prices. The VIX is a major component of STLFSI with a correlation of 0.9. Figure 2B charts the periods of the bad regime and STLFSI. Unsurprisingly, we can see that STLFSI is correlated with the regime (the regression R^2 is 0.45 and the slope coefficient is highly significant). But its indications tend to lag at the start of a bad regime. Our regime probability quickly shoots up at the start of crises, while STLFSI takes a longer time to reach a high value. Simply looking at implied volatility from option prices or from more comprehensive market indicators, such as STLFSI, provides less information than does our model with systemic jumps and international correlation breaks.

Asymmetries and Breaks in Correlation

A stylized fact about equity returns is that correlation is higher in bad markets than in good markets. We now show why our model captures correlation asymmetries better than alternative models. As seen in Table 2, the correlation between diffusion does not increase much between the good and bad regimes; the average correlations of diffusion are 0.63 in good markets and 0.68 in bad markets. On the other hand, the average correlation of jump sizes increases from 0.58 in the good regime to 0.91 in the bad regime. Both regimes are subject to jump risk, but jumps have a much larger negative expected size with higher correlation and volatility in the bad regime. These characteristics of jumps in the bad regime seem to match those of price changes after arrivals of extremely bad news. For instance, prices in all markets dropped sharply after the 9/11 attack. The collapses of Lehman Brothers and AIG brought great fears to the global markets and took major equity markets down markedly during late 2008. We refer to this drastic increase in correlation during the bad regime conditional on a jump arrival as correlation breaks.

To further illustrate, let's take the case of the Hong Kong market. In the good regime, the correlation of jump sizes between Hong Kong and the other countries is rather small (on average 0.37), providing good risk diversification benefits and illustrating the Chinese-diversification property of Hong Kong. But in the bad regime, systemic jumps affect all countries and jump sizes are highly correlated (on average 0.93 for HK). On the other hand the correlations of the diffusion components remain around 0.5 in both regimes. In bad markets, systemic jumps lead to a correlation break for Hong Kong, making it less attractive as a diversification investment.

As mentioned above, correlation for large absolute returns tends to be much higher for down-markets than for up-markets. But current models used in asset allocation fail to capture that stylized fact about extreme correlation. Longin and Solnik (2001) have proposed a famous *exceedance correlation* graph that gives the international correlation for returns that exceed a given threshold (for example larger than 5% per month or lower than -5%). We produce a similar graph for our data as shown in Figure 3. That figure shows results for actual data, our model, as well as simpler models with only jumps, only regime-switching or simple mean-variance. Although our model is not designed to optimize the fit to extreme correlation, it does match actual data very well. Having only jumps or only regime-switching provides a good match for positive exceedances but a very poor one for negative exceedances (crisis). Hence, we need both regime shifts and jumps to reflect what is happening in bad markets. To conclude, extreme price changes during financial crises cause asymmetries in extreme correlation, and jumps in the bad regime, which have the highest average correlation of 0.91 (Table 2), are the key success of capturing this stylized fact observed in the data.

Solnik and Watewai (2016) also studied how the most recent empirical models proposed in the literature fit actual extreme correlation. They considered three main classes of multivariate models: multivariate GARCH, factor copula, and multivariate factor stochastic volatility.[10] All these models drastically underestimate extreme correlation and would be poorly adapted to optimize asset allocation.

Asset Allocation Implications

We derive the multi-period global portfolio allocation with weekly rebalancing.[11] In each week we estimate a Bayesian probability q to be in the bad regime, and adjust the portfolio to account for current and future return distribution including asymmetries and breaks in correlation as modelled by regime-switching and jumps. We later test our portfolio model out-of-sample against various models, including the naïve $1/N$ model. But first let's look at some properties of the optimal allocations in-sample to understand the importance of using both regime-shifts and jumps.

It is well known that optimal asset allocations are very sensitive to the expected return assumptions and past sample mean returns are poor estimators of expected returns. Our focus is on risk-modeling not on forecasting returns. Hence we assume no knowledge of differential returns across countries and constrain country means to be equal. In the out-of-sample tests we use Bayesian shrinkage estimators for expected returns. Because most mutual funds are not allowed to short sell, but are allowed to borrow, we adopt this assumption in our analysis.[12]

Table 4 provides the optimal asset allocation weights which vary with the probability of the bad regime. The weights of the ten countries are grouped by regions: Asia-Pacific (AU, HK, and JP), Europe (FR, GE, SP, SW, and UK), and North America (CA and US). These are regional weights within the equity portfolio and hence sum to one. These regional weights suggest how investors should diversify across regions, and the total risky and risk-free weights suggest how much leverage positions should be taken. To understand the improvement brought by our modelling, we also report portfolio weights of the models without jumps or with a single regime. Figure 4 provides a graphic illustration.

[10] In particular Engle (2002), Oh and Patton (2017), Omori and Ishihara (2012).

[11] Investors maximize expected utility of the terminal wealth and have a constant relative risk aversion $\gamma = 5$, a typical assumption. The equity portfolio composition is quite stable for different levels of risk aversion. This value of γ generates strategies with volatility (standard deviation) being of an order of magnitude similar to that of passive global index strategies.

[12] The short-selling restriction rules out some hedge fund strategies arbitraging across markets, but these strategies are primarily based on country/market valuation (expected return), which is not our focus. We conducted the analysis without this restriction as well as with no leverage. The qualitative conclusions remain the same.

In our model, investors change their allocation based on the current probability of the bad market regime (q). Looking at our model, an investor, who is certain that the current regime is good ($q = 0$), will hold a leveraged position in equity. The equity allocation is 49% to America, 16% to Europe, and 35% to Asia-Pacific. That is not very different from market-capitalization weights. As q increases, the investment in the risky assets drops and Asia-Pacific replaces America, while Europe is stable within the risky portfolio. Investors keep holding risky assets until $q = 0.6$. As the expected return decreases with the higher probability of a crisis, risk focus becomes more important. That is achieved both by a reduced leverage and a higher allocation to the region providing the best diversification benefits (lower correlation). But when the probability of a bad regime looms large (say q over 0.4), the proportion of the risky assets gets smaller and a correlation break becomes more likely. This leads to a drastic reduction in the Asia-Pacific allocation as the correlation break is most pronounced for this region. The allocation to Europe goes up as some European countries are less sensitive to correlation breaks, while the America allocation keeps dropping. When the probability q goes over 0.6 there is no more allocation to equity.

In comparison, single-regime models, both without jumps (traditional MV) and with jumps, are static with constant allocation. Including jumps in single-regime models induces a reduction in the weight of regions more sensitive to jump risk, such as Asia-Pacific. The North-American weight is significantly increased relative to its MV weight.

Next, we compare the two-regime models without and with jumps. In the two-regime model without jumps, investors take less leverage for all probabilities q and they stop investing in equity for q around 0.4 (compared to 0.6 for our model). The equity allocations of both models are fairly similar when there is a large probability of a good regime (low q), but quite different when q is above 0.2.

This suggests that correlation asymmetries between the good and bad regimes have substantial impacts on the composition within the equity portfolio. Improved risk modeling (including jumps) allows a better differentiation between the regimes. It also allows for taking more aggressive positions in risky assets for a similar perceived risk level and reflects changes in diversification benefits as the probability of a crisis evolves.

Out-of-Sample Performance

A model with regime switching and jumps has a large number of parameters that may be subject to estimation errors. Testing the model out-of-sample is crucial. In a famous paper, DeMiguel, Garlappi and Uppal (2009) analyze out-of-sample portfolio performance of numerous mean-variance models, with and without taking into account estimation error, against the naive 1/*N* model.[13] They find that none of the models consistently outperforms the 1/*N* model and conclude that the gain from optimal diversification is more than offset by estimation error.

Expected returns play an important role in asset allocation weights but using past data to predict returns is fraught with estimation error in the mean. We use the Bayesian shrinkage method of DeMiguel, Martin-Utrera, and Nogales (2013) in which the estimate of the mean return of each asset is shrunk towards its grand mean (average of the means across assets). Our

[13] The 1/*N* model is an equally weighted portfolio in which each risky asset has the same portfolio weight and no weight is given to the risk-free asset.

analyses in this section rely on the shrinkage-mean model, but the results are qualitatively similar when means are not shrunk, or when means are constrained to be the same for all countries.

We now study the forward-looking results of the model against simpler models, including the robust 1/*N* model. Our out-of-sample period runs from January 2008 to December 2016, a total of nine years. This period covers various market crises (2008-2009, 2011, 2015-2016) and rallies (2009-2011, 2012-2014, 2016). We re-estimate the model every year using a seven-year rolling window. With the new estimates, we solve the dynamic portfolio optimization problem to obtain the optimal weights, which are functions of the market regime probability.[14] Table 5 provides the portfolio performance of five models: 1/*N*, one regime without jumps, one regime with jumps, two regimes without jumps, and two regimes with jumps (our model). We report the annualized values of mean and standard deviation of excess returns (over the Fed Fund rate), the Sharpe ratio and the maximum drawdown. We used a one-way transaction cost of 20 basis points (bp) for all trades. Our transactions are based on passive country indices for the ten largest equity markets, which would suggest lower costs. Back in 2010, Elkins/McSherry reported average overall transaction costs (commission, fee and market impact) below 20bp for the U.S. and even lower costs for several other countries (i.e. Japan and France). With the growth in trading platforms and dark pools, total global transaction costs for institutional managers have gone further down. Some global institutional investors report average overall transaction costs around 10bp.[15]

We find that our model with regime switching and jumps strongly outperforms the other four models in all aspects. Compared to the naïve 1/*N* strategy it provides a higher mean excess return (5.28% vs 1.36%) for a lower volatility (16.32% vs 21.83%). Hence its Sharpe ratio is much higher (0.324 vs 0.062). The single-regime models have a negative return. The model of two-regime without jumps is the second best but with a much lower excess return and Sharpe ratio. We also look at the passive MSCI World index made up of 23 developed markets as a proxy for market-capitalization-weighted index of our universe of the ten largest markets (well over 90% of the World index). The Sharpe ratio (0.171) is better than that of the 1/*N* strategy, but much lower than that of our model (0.324). As shown on the next figure, the return on MSCI World is better than the 1/*N* strategy because the US market (a big share of MSCI World) strongly outperformed most other markets in 2014-2016.

Figure 5 shows the accumulated wealth for all models. Our model shows a 67% increase in wealth while the two-regime model without jumps shows only an increase of 31%. The MSCI World shows an increase of 40% and the 1/*N* strategy shows an increase of 17%. On the loss side, our model has the lowest maximum drawdown of 29%, followed by the two-regime model without jump with the value of 37%. The other strategies have 55% or more maximum drawdown.

The strategy implies very frequent changes in asset allocation caused by any change in the regime probability. Hence the turnover and associated transaction costs are huge. We also test a more passive and realistic strategy. We keep the same allocation until there is a significant change in the regime probability (say more than 0.3 or 0.4) and then rebalance to the optimal weights given by the model. In other words, we only act when the probability of a bad regime

[14] Remember that investors do not know with certainty what the current regime is, but estimate the probability q of being in a bad/good regime.

[15] See for example: http://www2.westsussex.gov.uk/ds/cttee/pen/pen230714i4B.pdf

significantly increases or decreases. Table 6 gives the results of that strategy for a threshold of 0.4 in the regime probability change.[16] The first six rows give the result of the strategy for increasing one-way transaction costs from 0bp to 50bp. The last row gives again the results for the MSCI World index. The results for the other models are vastly inferior and not reported here. Results for a transaction cost of 20bp are better than before, as rebalancing is less frequent. Even with transaction costs of 50bp, our model strongly dominates the passive World index in all respects (excess return, volatility, Sharpe ratio and maximum drawdown). With a transaction cost of 50bp, the final wealth shows an increase of 79% compared to 40% for the World index. There are 54 rebalancing over the 9-year period or an average of 6 per year.

The reason for such a good performance is that we tend to detect crises very early. Regimes are persistent and the high correlation of observed jumps is a signal that a crisis has started. Hence the portfolio composition is adjusted to reflect the higher probability of a correlation break and leverage is reduced as well. We can see this clearly during the 2008-2009 and 2011 crises. On the other hand, a good regime is characterized by jumps of much smaller size and correlation. Shocks are easy to detect but it takes time to confirm that the process has become smoother. Our model takes time to estimate the return to a good regime.

It should be stressed that our approach is quite agnostic regarding expected returns. But returns tend to be low (negative) during crises, and breaks in correlation help us detect them quickly. The focus of this section was to illustrate the benefits of better risk modeling on some out-of-sample test covering nine recent years.

Conclusion

In the past decades, we have experienced numerous and sudden global market crises. Discontinuous shocks and sudden shifts in parameters have to be considered to improve risk management. In this article, we focus on risk modelling and propose directions to improve risk management and asset allocation in global equity portfolios. We do not intend to propose trading rules based on return forecasts as our model is very agnostic in terms of expected returns. Results are mostly illustrative of improvements brought by better treatment of asset correlation and volatility. But expected returns do vary with volatility and correlation, and our approach enjoys this added benefit.

All recent crises have led to a dramatic break in international correlation. Current risk management models often fail to depict it. The risk of a sudden break in correlation associated with higher volatility should be reflected in the optimal long-term asset allocation. Furthermore, market conditions (regimes) can change very quickly and are persistent; models of extreme correlation can detect such changes early and help adapt asset allocation tactically to changing market conditions. Our out-of-sample tests illustrate the importance of modelling correlation breaks.

Clearly asset prices do not follow simple unconditional lognormal distributions. A temptation is to use historical data to empirically fit a complex asymmetric distribution and numerically derive forward-looking asset allocation. A problem with these purely-empirical black boxes is that they provide little intuitive understanding of what is going on and how current

[16] Results for a lower probability threshold of 0.3 are fairly similar.

market conditions drive changes in the recommended asset allocation. On the other hand a regime-switching model with jumps is a fairly simple and intuitive model based on distributions with well-known properties. It can lend itself to a sensitivity analysis of portfolio weights to changes in model parameters. It also allows incorporating forward-looking belief/assumptions about parameters such as the size and probability of shocks or changes in volatility. In other words, it is sufficiently simple[17] and intuitive to be practically used by asset managers and integrated in the portfolio management process.

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Tables

Table 1. Summary Statistics of Weekly Log-Returns of Country Equity Indices

Country	Mean (% per week)	Std. Dev. (% per week)	Skewness	Excess Kurt.	Median (%)	Min (%)	Max (%)
AU	0.184	3.482	-1.598	13.354	0.522	-34.30	14.94
CA	0.123	3.183	-1.131	9.269	0.376	-26.05	17.76
FR	0.062	3.332	-1.008	6.545	0.384	-26.69	13.88
GE	0.087	3.580	-0.861	5.442	0.452	-26.06	15.20
HK	0.125	2.850	-0.281	2.463	0.287	-17.11	10.32
JP	0.044	2.676	-0.384	2.211	0.081	-16.40	11.02
SP	0.088	3.779	-0.903	4.924	0.300	-26.07	13.43
SW	0.111	2.690	-1.097	10.499	0.229	-23.91	13.12
UK	0.062	2.931	-1.215	12.315	0.306	-27.57	16.28
US	0.102	2.454	-0.881	7.740	0.232	-20.05	11.58

Note: The data cover the period from January 2001 to December 2016 (835 observations).

Table 2. Average Means, Standard Deviations, and Correlations of Return, Diffusion, and Jump Size

	Mean (% per week)		Standard Deviation (% per week)		Correlation	
	Bad regime	Good regime	Bad regime	Good regime	Bad regime	Good regime
Return	-0.310	0.257	4.471	2.347	0.758	0.627
Diffusion	0.166	0.442	3.331	2.044	0.680	0.634
Jump Size	-2.741	-1.104	6.474	2.512	0.905	0.576

Table 3. Regime and Jump Statistics

	Probability of staying in the same regime	Expected duration (weeks)	Jump arrival rate (per week)	Average of jump size		
				Mean (% per week)	S.D. (% per week)	Correlation
Bad regime	0.961	25.49	0.174	-2.741	6.474	0.905
	(0.015)	(9.73)	(0.035)	(1.279)	(0.931)	(0.038)
Good regime	0.985	65.72	0.168	-1.104	2.512	0.576
	(0.006)	(24.84)	(0.016)	(0.345)	(0.255)	(0.083)

Note: Standard errors are given in the parentheses.

Table 4. Optimal Portfolio Weights

	One regime without jumps	Two regimes without jumps							
		$q = 0$	$q = 0.1$	$q = 0.2$	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$	$q \geq 0.7$
Risky-asset port									
Asia-Pacific	0.538	0.354	0.446	0.527	0.437	0.000	-	-	-
Europe	0.188	0.186	0.158	0.183	0.563	1.000	-	-	-
North America	0.274	0.460	0.396	0.290	0.000	0.000	-	-	-
Total risky wgt	0.393	1.515	0.862	0.431	0.124	0.020	0.000	0.000	0.000
Risk-free wgt	0.607	-0.515	0.138	0.569	0.876	0.980	1.000	1.000	1.000
	One regime with jumps	Two regimes with jumps (our model)							
		$q = 0$	$q = 0.1$	$q = 0.2$	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$	$q \geq 0.7$
Risky-asset port									
Asia-Pacific	0.505	0.353	0.431	0.482	0.531	0.594	0.448	0.007	-
Europe	0.167	0.157	0.151	0.159	0.198	0.367	0.552	0.993	-
North America	0.329	0.490	0.418	0.360	0.271	0.038	0.000	0.000	-
Total risky wgt	0.554	2.168	1.230	0.751	0.425	0.192	0.076	0.012	0.000
Risk-free wgt	0.446	-1.168	-0.230	0.249	0.575	0.808	0.924	0.988	1.000

Table 5. Out-of-Sample Portfolio Performance

	Mean excess return (% per year)	Standard deviation (% per year)	Sharpe ratio	Maximum drawdown (%)
$1/N$	1.36	21.83	0.062	55.31
One regime without jumps	-4.48	17.46	-0.257	57.41
One regime with jumps	-4.48	23.48	-0.191	67.28
Two regimes without jumps	2.59	17.16	0.151	36.99
Two regimes with jumps	5.28	16.32	0.324	29.37
MSCI World index	3.36	19.69	0.171	54.70

Table 6. Out-of-Sample Portfolio Performance with Infrequent Rebalancing

Transaction cost (basis points)	Mean excess return (% per year)	Standard deviation (% per year)	Sharpe ratio	Maximum drawdown (%)
0	8.89	12.24	0.726	17.73
10	8.33	12.26	0.679	18.33
20	7.76	12.28	0.632	19.06
30	7.19	12.30	0.584	19.78
40	6.62	12.34	0.537	20.50
50	6.05	12.37	0.489	21.21
MSCI World index	3.36	19.69	0.171	54.70

Figures

Figure 1. Probabilities of Bad Regime

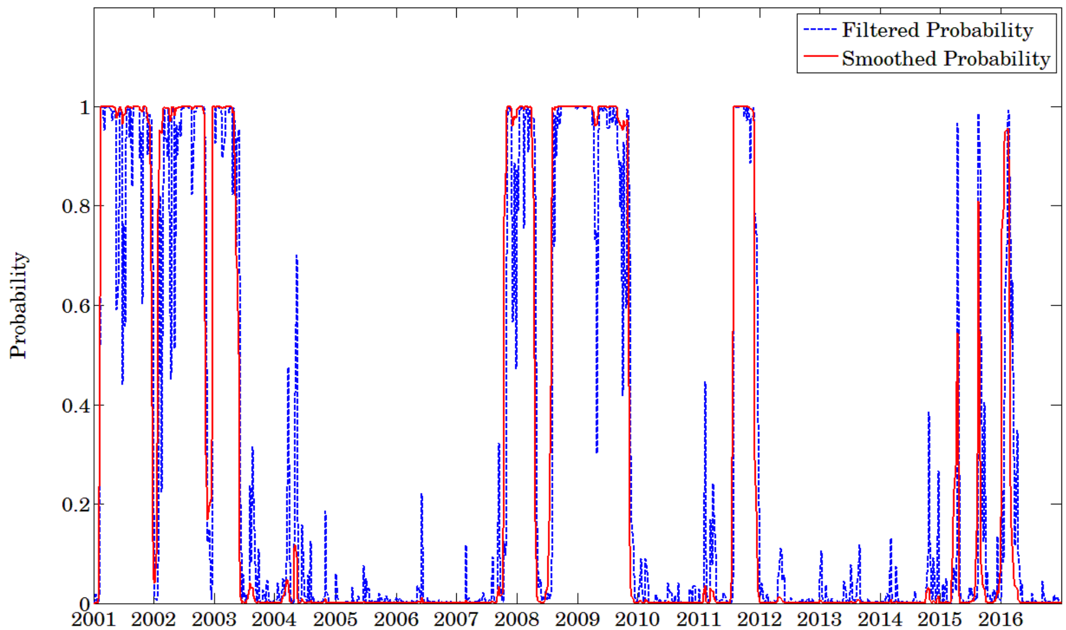
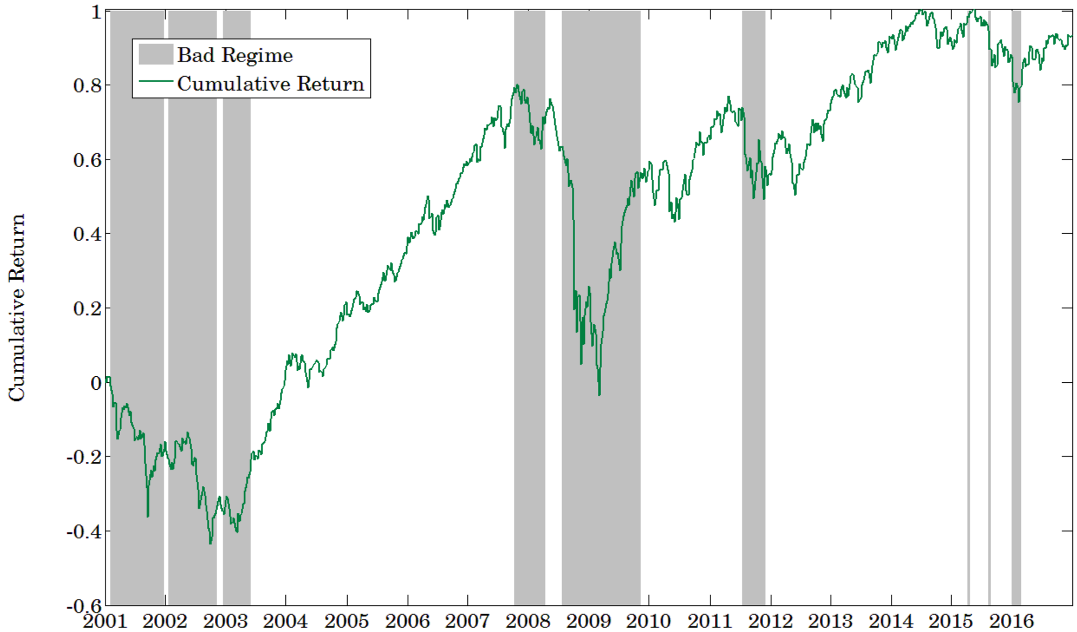


Figure 2. Global Portfolio, STLFSI, and the Bad Regime Periods

A. Periods of the Bad Regime and Cumulative Return of Equally Weighted Portfolio



B. Periods of the Bad Regime and STLFSI

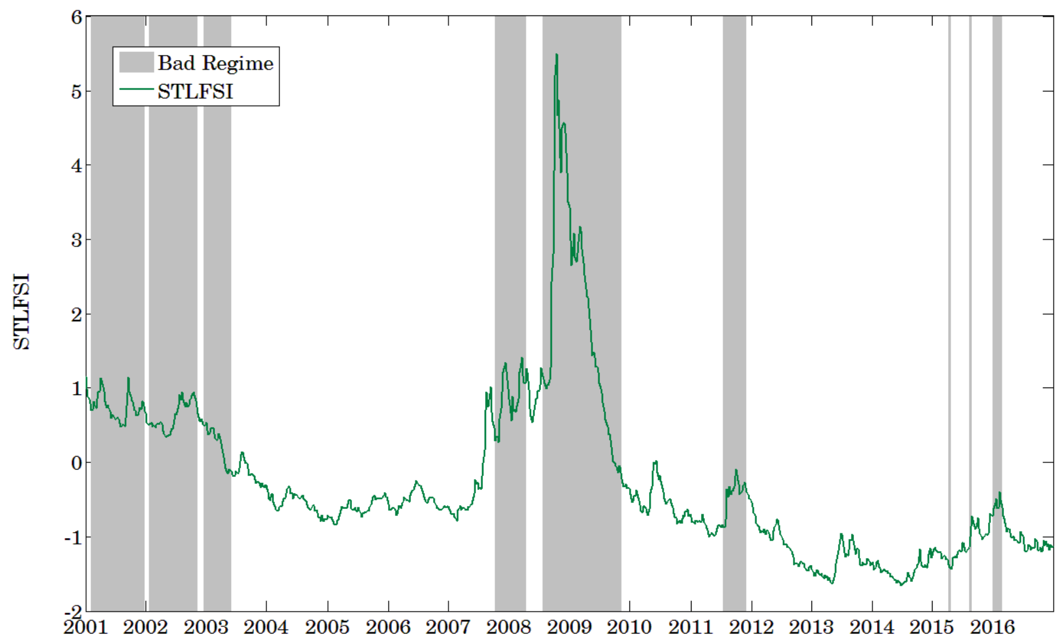


Figure 3. Exceedance Correlations

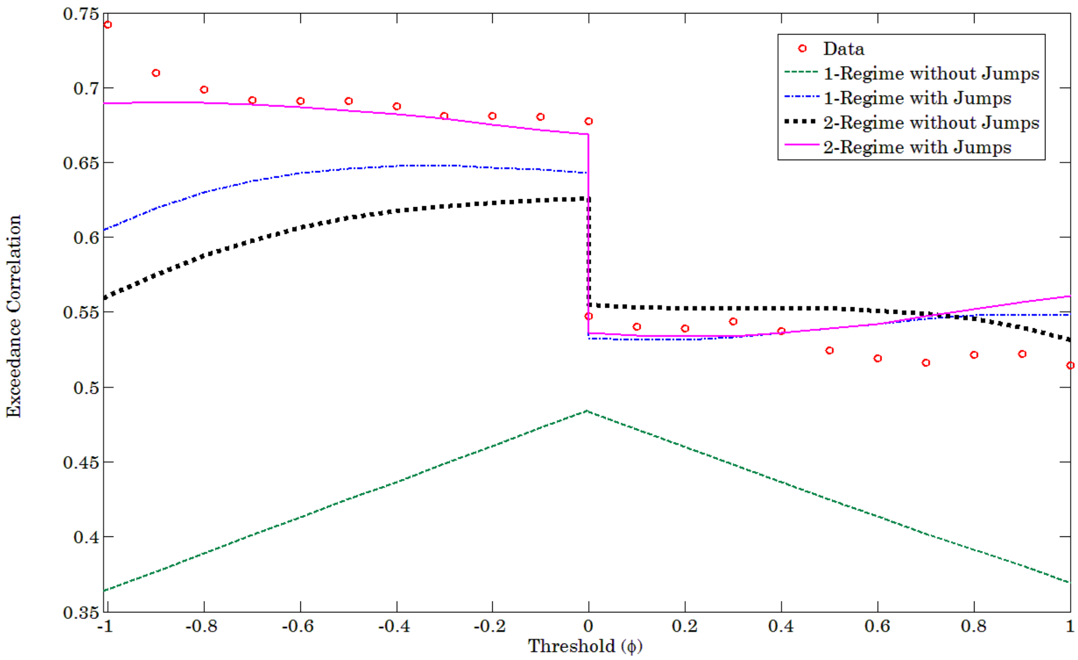
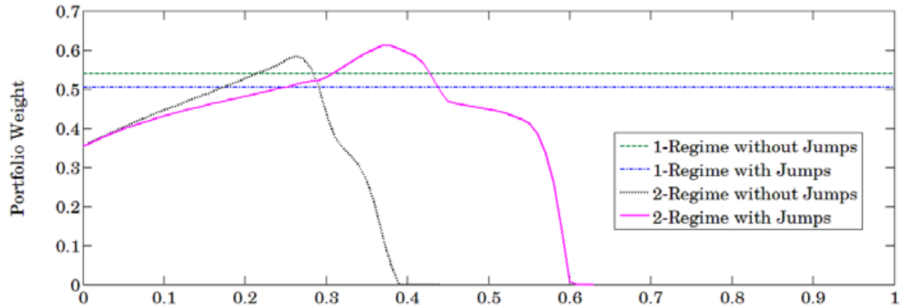
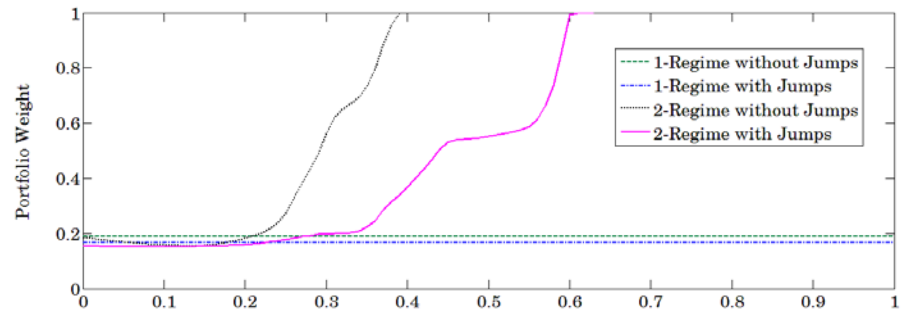


Figure 4. Optimal Portfolio Weights

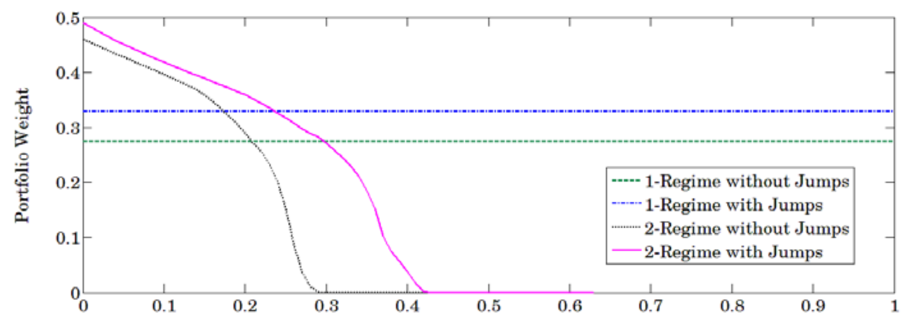
A. Asia-Pacific Weights



B. Europe Weights



C. North America Weights



D. Total Risky Weights

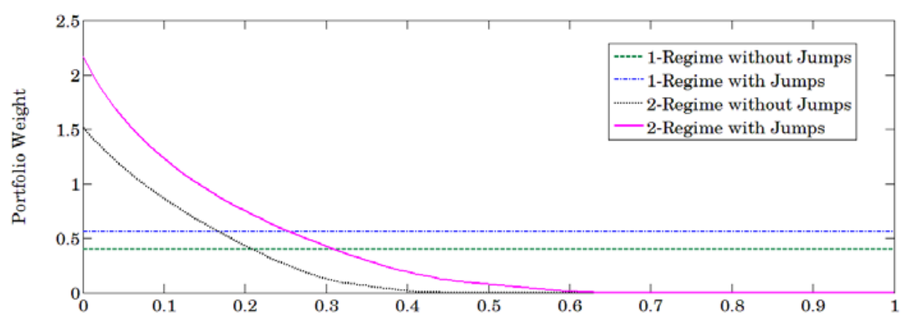


Figure 5. Out-of-Sample Wealth Processes