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# DOES EXTREME CORRELATION MATTER IN GLOBAL EQUITY ASSET ALLOCATION?

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*Global asset allocation provides risk diversification. But international market correlation increases sharply during global crises and diversification benefit disappears when it is most needed. We model these correlation breaks and derive the asset allocation implications. The model can quickly detect crises and suggests adapting allocation for changing correlation and volatility, as the crisis probability evolves. The out-of-sample results for ten major equity markets over 2008–2016 show significant improvements in the Sharpe ratio and maximum drawdown over mean–variance, fat-tail distribution, passive indices, and $1/N$ rule. A benefit of the model is that it is conceptually intuitive and amenable to simple implementation in asset allocation and risk management.*

Since Markowitz (1952), practitioners have based their optimization of global equity allocation on mean–variance (MV) analysis or variants thereof. Undoubtedly, formulating expected returns (risk premia) is a crucial part of a successful asset allocation. Returns are hard to predict and risk should be managed. However, the risk side of optimizers has remained quite simple and static. Risk diversification has been a major motivation for global equity allocation. But the past decades have shown that equity markets go through prolonged periods of global crisis where returns are low while volatility and international correlation are high. With high correlation, the benefits of international diversification nearly disappear. To simplify, markets globally go through periods of good and bad regimes with very different volatility and correlation characteristics. The risk level and the benefits from international diversification vary markedly. But developing investment management tools that reflect time-varying correlation has proven difficult, if not impossible, and is the motivation of this paper.

Past crises have taught us that breaks in correlation could lead to huge losses in global portfolios, as illustrated by LTCM in the 1998 crisis or the

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2008–2009 crisis. Portfolios that appeared well diversified exhibited huge drops in value because most of their investments became highly correlated. To illustrate why correlation is a key driver of risk, let us consider a portfolio spread evenly between ten uncorrelated investments, each with a volatility of 10%. Then the portfolio volatility is 3.16%. If markets become more volatile with a volatility doubling from 10% to 20%, the portfolio volatility goes up to 6.32% as long as the correlation remains null. But if correlation breaks and shoots up to one, then the portfolio volatility goes up to 20%. The increase from 6.32% to 20% is solely explained by the correlation break. One of the stylized facts of global equity markets is that returns tend to be more correlated when volatilities are high or when the markets go down. Extreme correlation, i.e. correlation for large absolute returns, tends to be much higher for down-markets than for up-markets, as shown by Longin and Solnik (2001) and extensive subsequent research.[1] This could not happen if return distribution followed a normal distribution. While time-variation in volatility has been extensively studied, research on time-variation in international correlation is less developed and mostly descriptive.[2] As stressed by Forbes and Rigobon (2002), there can also be a spurious increase in correlation due to increase in volatility if correlation and volatility are not modeled carefully, as we do in our model.

It appears that there are shifts in market conditions, especially during crises. Unfortunately, we are never sure in which regime the market will be in the next few months. Asset allocation optimization and risk management should reflect the time-variation in return distributions. Time-variation affects both strategic and tactical allocation. Simply basing the strategic allocation on some long-term risk measures does not do the trick. One must reflect the risk of crises as we do here. The long-term strategic allocation should take into account the risk of a crisis appearing in the future with time-varying risk parameters, in the spirit of Merton (1973) dynamic modeling of uncertain shifts in investment opportunities. The tactical asset allocation should reflect not only changes in expected returns but also the probability of a change in risk parameters.

Recent literature tries to propose practical asset allocation strategies that protect portfolios against financial crises. For example, Xiong and Idzorek (2011) propose to use Conditional Value at Risk (CVaR), instead of variance, as it takes skewness and kurtosis into account, and show that M-CVaR improves the portfolio performance over MV during the 2008 crisis. However, as pointed out by Greiner (2012), a more interesting question, which is still left unanswered, is whether the extreme correlation or fat tail is the main contributor of the improvement.[3] Wang *et al.* (2012) propose to time the market by reducing/increasing the risk limit in M-CVaR when a CVaR forecast is high/low. But market timing has not been linked to shifts in extreme correlations. Kritzman *et al.* (2012) propose another market timing strategy by tilting the allocation weights based on regime shifts of market turbulence, inflation, and economic growth variables. Their suggestion on the direction of the tilts relies mostly on risk premium (i.e. return) rather than on volatility or correlation. It seems the increased correlation during bad markets is somewhat left out in these strategies despite being a well-established empirical fact. One possible exception is Kritzman *et al.* (2011) who develop the absorption ratio based on the principal component approach, which is closely related to correlation. They propose to use a shift in the ratio as a market-timing signal for changing allocation between stocks and bonds. However, as the ratio is designed to measure systemic risk, it is not clear how to use the ratio to allocate weights between risky assets.

Deriving simple allocation rules from complex dynamic models with time-varying risk parameters has been proven technically impossible. In this paper, we rely on an asset allocation model introduced by Solnik and Watewai (2016) that is tractable and provides interesting applications to global equity portfolios. They use two simple and well-known return distributions, normal and Poisson jump process, to model returns with global shocks. They further add Markov regime switching, where the global market conditions switch between a good and a bad regime. The properties of Poisson jumps and regime switching have been extensively studied in the finance literature, and the concepts are simple and intuitive. This relatively simple modeling choice allows stylized facts such as fat tails, skewness, and time-varying volatility and correlations. It can also model crises with increased volatility and correlation breaks. We can empirically estimate the return correlations for good and bad regimes, and correlations for normal returns and for unexpected shocks. So we can get a good intuitive understanding of how changes in market conditions and risk parameters, especially correlations, affect optimal asset allocation as the return distributions are simple and intuitive. This is quite different from assuming some ad-hoc complex distributions with parameters dictating the shape of the distribution.

Another advantage of this model is that we can estimate the model and derive the optimal asset allocation rules for a reasonably large number of countries. It is also easy to implement as the allocation rules are simply a function of the probability being in a good/bad regime.[4] In this paper, we show that modeling extreme correlation, i.e. the level of correlation when market returns take extreme values (positive or negative), yields a significant improvement in global equity portfolios. The benefit to practitioners is twofold. We provide a powerful risk management tool that is yet simple enough and intuitive. We offer asset-allocation rebalancing rules to adjust to changes in market conditions. Hence our model is both descriptive and normative. In its most simple application the model relies on a prior on the probability of being in a good/bad regime. The model is complex, but once programmed it will easily generate an updated regime probability which is central to rebalancing the asset allocation. As the reasons for changes in that probability are clear and intuitive, one could even incorporate subjective beliefs of the asset manager in a Bayesian manner.

We provide a brief description of the model in Section 1. We use the full-sample estimation results to study the return characteristics in Section 2. The analyses are based on the ten largest equity markets over the period 2001–2016. We focus on the correlations for small and large returns and how they change as markets switch between good and bad regimes. The results show that the model with regime switching and jumps yields a significant improvement on fitting extreme correlations. In Section 3, we describe the effect of time-variation in correlation and volatility on the optimal portfolio. In Section 4 we look at the predictive value of the model. We dynamically estimate the optimal asset allocation using only past data and check its future performance to illustrate the benefit of a better treatment of changing correlations. The results show that our model is able to quickly detect starts of crises, and suggests better leverage position and allocation among the markets. As a result, it has much lower risk and higher return out-of-sample compared to the other strategies. We conclude by looking at practical implications for global asset allocation. Technical details are provided in the Appendix.

## 1 The model

We now describe our model with regime switching and jumps based on Solnik and Watewai (2016). First, let us provide some intuition for jumps and regime switching. A simple asset price model usually assumes that returns are normally distributed. However, asset prices are subject to sudden changes (shocks) causing the return distribution to have fat tails. Such unusual price changes do not occur in every period, but when they come, the price "jumps." Equity markets go through good times and bad times. During good times, stock prices tend to go up and returns tend to have low volatility and international correlation, while the prices tend to go down and returns tend to have high volatility and correlation during bad times. The change in the characteristics of the return distribution is called "regime switching." In our model, country returns are driven by a normal distribution component ("normal") and random global shocks following a Poisson jump process ("jump"). In other words, the return of a single market $i$ at time $t$ is $R_{i,t}$ given by:

$$R_{i,t} = Normal_{i,t} + Jump_{i,t}, \quad t = 1, 2, \ldots \quad (1)$$

where *Normal* follows the normal distribution and *Jump* follows a Poisson jump process.

The properties of the normal distribution and Poisson jump process are well known and have been studied extensively.[5] Each distribution can be described by a few parameters that we estimate empirically. In particular, the normal component can be described by means, variances, and correlations. To describe the distribution of jumps, we need two quantities. The first is the jump frequency to describe how often a jump occurs on average. The average number of jumps per unit time describes it. For example, if we expect to see two jumps every quarter on average, the average number of jumps is eight per year. However, the actual arrival times of jumps as well as the actual number of jumps in a given year are random. The second is the jump size to describe how large a jump is. The jump size is random and correlated across countries. The means, variances, and correlations describe the jump size distribution.

We assume that all jumps are systemic or global, so that *all* markets are affected in some random magnitude when a jump occurs. As mentioned earlier, jumps represent unexpected shocks in the global market. These shocks could be due to a global panic from a market crash like the collapse of Lehman Brothers, or a sharp response to certain events like a sudden surge in the oil price or an unexpected interest rate hike. On the other hand, with the absence of jumps, the returns simply follow the normal distribution. Loosely speaking, the normal component models the return distribution in the absence of jumps.[6] As global markets do experience unanticipated large shocks from time to time, modeling jumps clearly enhances portfolio risk management capability as we show below. Further technical details are given in the Appendix.

There are two different regimes based on the Markov regime-switching process as introduced by Hamilton (1989). Regime switching has been used extensively in the finance literature.[7] The basic idea is that market conditions are driven by one of two market regimes (typically a good and a bad or crisis one). The regimes are unobservable, but we can estimate the probability of being in a given regime. The parameters of the return distributions for both normals and jumps in (1) are different in each regime. So it allows us to have different means, volatilities, and correlations for good and bad regimes. In fact, in the empirical part we find that regimes are persistent and one regime (the "*bad*" regime) has much lower mean return and much higher volatility and correlation than the other regime (the "*good*" regime).

To summarize, the return in this model comes from two components, normals and jumps, and each component is subject to the same global regime shifts. At each time period, we observe returns and infer the probability of being in the bad or crisis regime. Although each economic/financial crisis is somewhat different from the previous ones and is triggered by different and somewhat-unpredictable events, financial price behavior tends to be fairly similar under crisis: lower risk tolerance, liquidity crisis affecting prices to drop, higher volatility (typically 2 to 3 times), higher correlation across all asset classes (correlation break), and very rapid shift from good to bad regime. Such stylized facts can be modeled by the jump component that has more negative mean and higher volatility and correlation in the bad regime than in the good regime. With that asymmetric jump distribution, the model should be able to quickly increase the bad-regime probability soon after the crisis starts and thus helps us adjust the portfolio to avoid massive losses. That is, jumps and regime switching together provide an effective way of detecting crises. We confirm this modeling benefit in the out-of-sample result below. Technical details about the model and estimation method are provided in the Appendix.

## 2 Model estimation

### 2.1 Data

We use MSCI weekly total return data in USD for the ten largest investable markets: Australia (AU), Canada (CA), France (FR), Germany (GE), Hong Kong (HK), Japan (JP), Spain (SP), Switzerland (SW), the United Kingdom (UK), and the United States (US). Our sample covers well over 90% of the market capitalization of the MSCI World index. China is a closed market but many Chinese companies are listed on the Hong Kong exchange. Weekly data are used to alleviate the nonsynchronicity problem of daily data caused by time difference. We use the Friday-close to Friday-close return data from January 2001 to December 2016 (835 observations) as the weekly total returns are available starting from January 2001. Monthly data are available from an earlier date but is not consistent with our primary objective to quickly detect changes in regime and correlation.[8] Despite the fairly short time

**Table 1** Summary statistics of weekly returns of country equity indices.

| Country | Mean (% per week) | Standard deviation (% per week) | Skewness | Excess Kurt. | Median (%) | Min (%) | Max (%) |
|---|---|---|---|---|---|---|---|
| AU | 0.184 | 3.482 | −1.598 | 13.354 | 0.522 | −34.30 | 14.94 |
| CA | 0.123 | 3.183 | −1.131 | 9.269 | 0.376 | −26.05 | 17.76 |
| FR | 0.062 | 3.332 | −1.008 | 6.545 | 0.384 | −26.69 | 13.88 |
| GE | 0.087 | 3.580 | −0.861 | 5.442 | 0.452 | −26.06 | 15.20 |
| HK | 0.125 | 2.850 | −0.281 | 2.463 | 0.287 | −17.11 | 10.32 |
| JP | 0.044 | 2.676 | −0.384 | 2.211 | 0.081 | −16.40 | 11.02 |
| SP | 0.088 | 3.779 | −0.903 | 4.924 | 0.300 | −26.07 | 13.43 |
| SW | 0.111 | 2.690 | −1.097 | 10.499 | 0.229 | −23.91 | 13.12 |
| UK | 0.062 | 2.931 | −1.215 | 12.315 | 0.306 | −27.57 | 16.28 |
| US | 0.102 | 2.454 | −0.881 | 7.740 | 0.232 | −20.05 | 11.58 |

This table provides summary statistics of weekly returns of ten MSCI country equity indices, including Australia (AU), Canada (CA), France (FR), Germany (GE), Hong Kong (HK), Japan (JP), Spain (SP), Switzerland (SW), the United Kingdom (UK), and the United States (US). The returns are computed from the total return indices in US dollars. The data cover the period from January 2001 to December 2016 (835 observations).

period, it ensures good data quality, reflects the market openness to foreign investments, and, importantly, covers many market cycles.

Table 1 shows the summary statistics of weekly returns of each index in % per week. All index returns have negative skewness, confirming the asymmetric distributions of returns. The minimum and maximum values show that negative shocks could be much larger than positive shocks. Over the period, the most severe minimum weekly return is −34.30%, while the largest positive weekly return is +17.76%. Such extreme returns, say beyond three standard deviations, have an extremely small probability of occurring under the normal distribution (0.27%), but we do observe them repeatedly during crises. Jumps can help explain those extreme returns. Excess kurtosis (fat tails) is very large and ranges from 2.21 for Japan to 13.35 for Australia. Such high kurtosis could be explained by jumps and nonstationarity. Regime switching and jumps can address nonstationarity in return distributions.

### 2.2 *Estimation results*

We do not report detailed parameter estimates for each country, but simply their averages across the ten countries.[9] We also performed all kinds of statistical tests not reported here.[10] Table 2 shows the average means, standard deviations and correlations of the return, normals, and jump size. Relative to the good regime, the bad regime is characterized by lower return (−0.31% per week vs +0.26%), higher volatility (4.47% per week vs 2.35%), and higher correlation (0.76 vs 0.63). The big difference between the two regimes in terms of means, volatility, and correlation is induced by jumps.

A crucial result is that regimes are persistent. Markets go through prolonged periods of good and bad regimes. Figure 1 shows the filtered (dashed line) and smoothed (solid line) probabilities of being in the bad regime. The probability to stay in the same regime is high (above 95%), and the expected regime duration is long, as shown in Table 3. The expected duration is 25 weeks for the bad regime and 66 weeks for the good regime. Detecting a regime shift at an early stage enables the investors to adjust their portfolios for the new lengthy regime in which international correlation and volatility, among others, change drastically. A large negative jump can lead to a rapid transition to the bad regime, and the regime is persistent, so a crisis can be quickly detected. The benefit of

**Table 2** Average means, standard deviations, and correlations of return, normals, and jump size.

| | Mean (% per week) | | Standard deviation (% per week) | | Correlation | |
|---|---|---|---|---|---|---|
| | Bad regime | Good regime | Bad regime | Good regime | Bad regime | Good regime |
| Return | −0.310 | 0.257 | 4.471 | 2.347 | 0.758 | 0.627 |
| Normal | 0.166 | 0.442 | 3.331 | 2.044 | 0.680 | 0.634 |
| Jump Size | −2.741 | −1.104 | 6.474 | 2.512 | 0.905 | 0.576 |

This table provides the averages of the estimated means, standard deviations, and correlations of returns, normals, and jump sizes. The averages are provided for each regime. The estimates are based on the weekly data of the ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US) from January 2001 to December 2016.

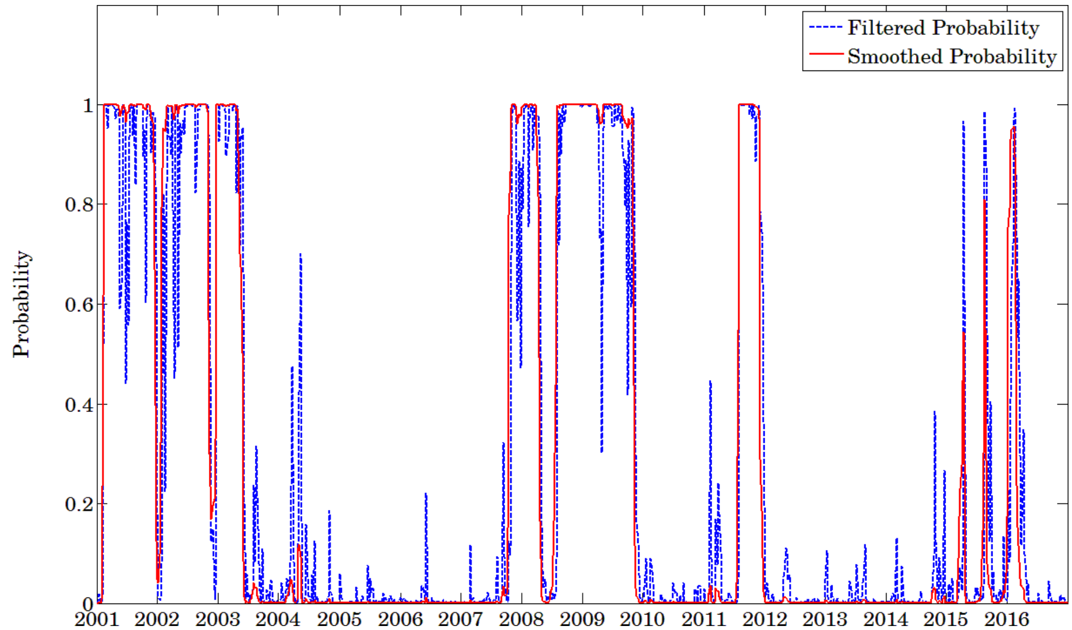



**Figure 1** Probabilities of bad regime.

This figure shows the filtered (dashed line) and smoothed (solid line) probabilities of being in the bad regime. The estimates are based on the weekly data of the ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US) from January 2001 to December 2016.

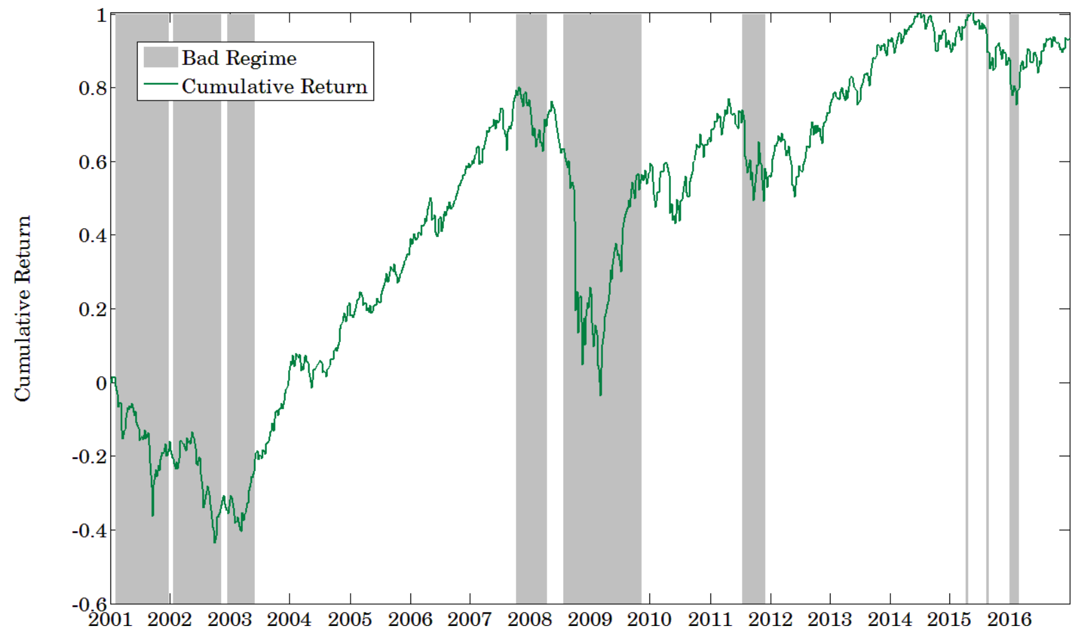
**Table 3** Regime and jump statistics.

| | | | | Average of jump size | | |
|---|---|---|---|---|---|---|
| | Probability of staying in the same regime | Expected duration (weeks) | Jump arrival rate (per week) | Mean (% per week) | Standard deviation (% per week) | Correlation |
| Bad regime | 0.961 | 25.49 | 0.174 | −2.741 | 6.474 | 0.905 |
| | (0.015) | (9.73) | (0.035) | (1.279) | (0.931) | (0.038) |
| Good regime | 0.985 | 65.72 | 0.168 | −1.104 | 2.512 | 0.576 |
| | (0.006) | (24.84) | (0.016) | (0.345) | (0.255) | (0.083) |

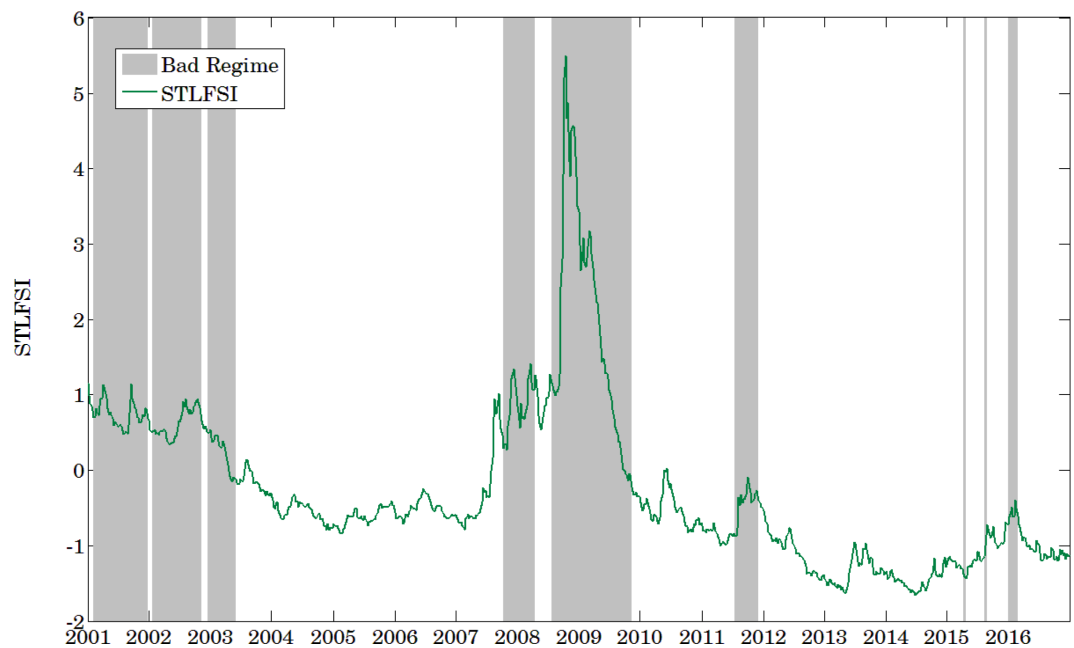
This table provides the estimates of weekly transition probabilities, expected durations, jump arrival rates, and jump size statistics of both regimes. Standard errors are given in the parentheses. The expected duration for a given regime is computed from the inverse of the rate at which the regime will shift to another regime. The estimates are based on the weekly data of the ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US) from January 2001 to December 2016.

early detection is confirmed by the out-of-sample results below.

We can gain some insights on regimes by looking at Figure 2A which charts periods of bad regime (gray) and the returns on an equal-weight global portfolio. The model quickly detects the start of a bad regime. This is true in all bad periods detected in the model. The increased correlation and volatility due to persistent big jumps allow for an early detection of bad regimes. A bull market is characterized by much smoother returns, and smoothness takes time to be confirmed. Furthermore, in the early stage of recovery, returns keep being quite correlated with fairly large volatility. Hence our model fails to quickly detect return to




A. Periods of the bad regime and cumulative return of equally weighted portfolio




B. Periods of the bad regime and STLFSI

**Figure 2** Global portfolio, STLFSI, and the bad regime periods.

This figure shows the cumulative return of an equally weighted portfolio of the ten countries (A) and STLFSI (B) in solid lines. The bad regime periods are represented by the shaded areas. The estimates of regime periods are based on the weekly data of the ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US) from January 2001 to December 2016.

a bull market. On the other hand, large negative systemic shocks easily signal a bad regime.

Changes in volatility help detect regimes but international correlation improves detection. This is also illustrated in Figure 2B. There are few high-frequency indicators of financial crisis, and they are domestic ones, primarily from the US. We focus on the STLFSI (Saint Louis Fed's Financial Stress Index), which is a broad weekly index of financial stress, derived from a principal components analysis of 18 financial indicators based on market prices. The VIX is a major component of STLFSI with a correlation of 0.9. Figure 2B charts the periods of the bad regime and STLFSI. Unsurprisingly, we can see that STLFSI is correlated with the regime (the regression $R^2$ is 0.45 and the slope coefficient is highly significant). But its indications tend to lag at the start of a bad regime. Our regime probability quickly shoots up at the start of crises, while STLFSI takes a longer time to reach a high value. Simply looking at implied volatility from option prices or from more comprehensive market indicators, such as STLFSI, provides less information than does our model with systemic jumps and international correlation breaks.

## 2.3 *Asymmetries and breaks in correlation*

A stylized fact about equity returns is that correlation is higher in bad markets than in good markets. We now show why our model captures correlation asymmetries better than alternative models. As seen in Table 2, the correlation between normals in the bad regime does not increase much from that in the good regime; the average correlations of normals are 0.63 in good markets and 0.68 in bad markets. On the other hand, the average correlation of jump sizes increases from 0.58 in the good regime to 0.91 in the bad regime. Both regimes are subject to jump risk, but jumps have a much larger negative expected size with higher correlation and volatility in the bad regime. These characteristics of jumps in the bad regime seem to match with those of price changes after arrivals of extremely bad news. For instance, prices in all markets dropped sharply after the 9/11 attack. The collapses of Lehman Brothers and AIG brought great fears to the global markets and took major equity markets down markedly during late 2008. We refer to such a drastic increase in correlation during the bad regime conditional on a jump arrival as a correlation break.

To further illustrate, let us take the case of the Hong Kong market. In the good regime, the correlation of jump sizes between Hong Kong and the other countries is rather small (on average 0.37), providing good risk diversification benefits and illustrating the Chinese-diversification property of Hong Kong. But in the bad regime, systemic jumps affect all countries and jump sizes are highly correlated (on average 0.93 for HK). On the other hand the correlations of the normal components remain around 0.5 in both regimes. In bad markets, systemic jumps lead to a correlation break for Hong Kong, making it less attractive as a diversification investment.

As mentioned above, correlation for large absolute returns tends to be much higher for down-markets than for up-markets. But models currently used in asset allocation fail to capture that stylized fact about extreme correlation. Longin and Solnik (2001) have proposed an *exceedance correlation* graph that gives the international correlation for returns that *exceed* a given threshold (for example, larger than 5% or lower than −5%). This type of graph has since been extensively used in the correlation literature, as it gives a good visual picture of the changes in correlation for different market conditions, including extremes, and does not suffer from statistical biases. It is well established from the literature that empirical exceedance correlations increase as the returns become more negative. We produce a

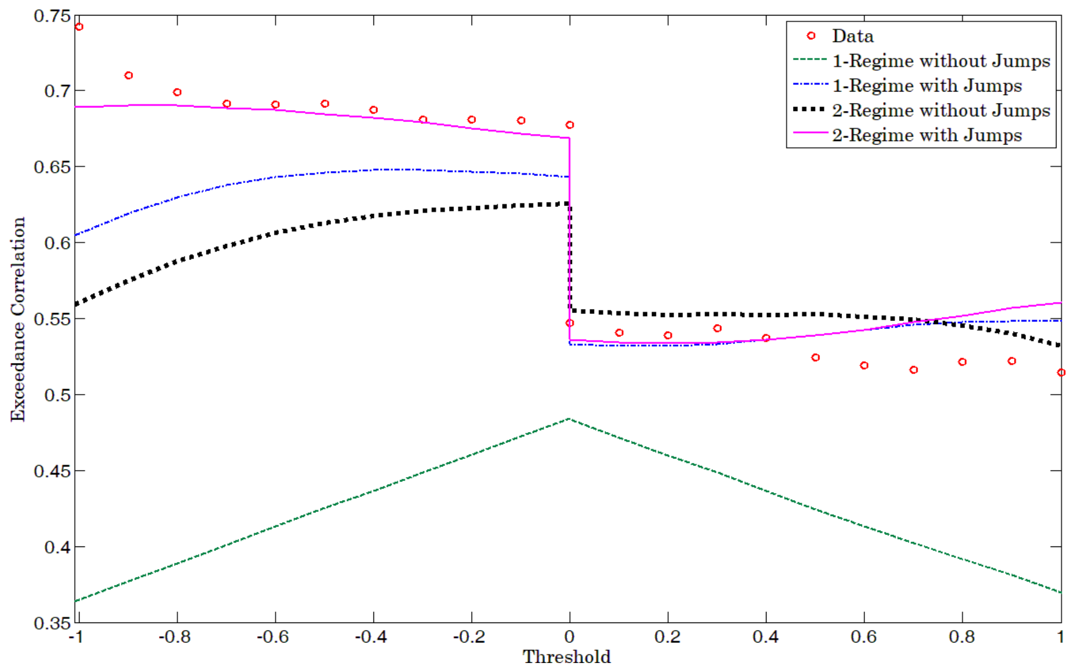



**Figure 3** Exceedance correlations.

This figure shows the average exceedance correlations from 45 country pairs. It contains the average exceedance correlations computed from observed data (circles), one-regime model without jumps (dashed line), one-regime model with jumps (dot-dashed line), two-regime model without jumps (dotted line), and two-regime model with jumps (solid line). The estimates are based on the weekly data of the ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US) from January 2001 to December 2016.

similar graph for our data as shown in Figure 3. That figure shows the correlation at different threshold levels for actual data, our model, as well as simpler models with only jumps, only regime switching and simple mean–variance. Returns are standardized by their standard deviation. If we take the example of actual data (circles), we see that the correlation is much higher for negative than positive returns, and it increases for very negative returns. For example, the correlation is 0.69 for return of $-0.5$ or more negative and 0.52 for returns of $+0.5$ or higher. Although our model is not designed to optimize the fit to extreme correlation, it does match actual data very well. Having only jumps or only regime switching provides a good match for positive exceedances but a very poor one for negative exceedances (crisis). Hence, we need both regime shifts and jumps to reflect what is happening in bad markets. To conclude, extreme price changes during financial crises cause asymmetries in extreme correlation, and jumps in the bad regime, which have the highest average correlation of 0.91 (Table 2), are the key success of capturing this stylized fact observed in the data.

## 3 Effect of risk modeling on optimal allocation

In this section we provide a description of the effects of better risk modeling on the optimal allocation of global portfolios. We derive the multi-period global portfolio allocation based on expected utility maximization with weekly rebalancing.[11] In each week we estimate a Bayesian probability $q$ to be in the bad regime, and adjust the portfolio to account for current and future return distribution including asymmetries and breaks in correlation as modeled by regime switching and jumps.[12] We later test our portfolio model out-of-sample (predictive) against various models, including the naïve $1/N$ model. But first let us look at some properties of the optimal allocations in-sample to understand the importance of using both regime shifts and jumps.

The effect of the country mean returns on optimal portfolio weights is simple: a country with a higher mean return deserves a higher portfolio weight, given all other things equal. However, the effect of risk modeling on asset allocation between countries is far more complex, especially when there are regime shifts, and hence it needs an investigation. To get a clear picture on the risk modeling effect, we remove the effect of the difference in the country means by re-estimating the model with the restriction that the mean return is the same for every country. We then study the optimal portfolio weights implied from the re-estimated model. Note that we allow the means in the good and bad regimes to be different. So when the probability of the bad regime increases, it is likely that the weights in the risky assets will decrease due partly to the lower mean in the bad regime. However, the change in the *relative* allocation between the countries is not affected by the difference in the country mean, as they are always equal. This allows us to focus on the effect of risk modeling on the optimal relative allocation

**Table 4** Optimal portfolio weights.

| | One regime without jumps | Two regimes without jumps | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $q = 0$ | $q = 0.1$ | $q = 0.2$ | $q = 0.3$ | $q = 0.4$ | $q = 0.5$ | $q = 0.6$ | $q \geq 0.7$ |
| Risky-asset port | | | | | | | | | |
| Asia-Pacific | 0.538 | 0.354 | 0.446 | 0.527 | 0.437 | 0.000 | — | — | — |
| Europe | 0.188 | 0.186 | 0.158 | 0.183 | 0.563 | 1.000 | — | — | — |
| North America | 0.274 | 0.460 | 0.396 | 0.290 | 0.000 | 0.000 | — | — | — |
| Total risky wgt | 0.393 | 1.515 | 0.862 | 0.431 | 0.124 | 0.020 | 0.000 | 0.000 | 0.000 |
| Risk-free wgt | 0.607 | −0.515 | 0.138 | 0.569 | 0.876 | 0.980 | 1.000 | 1.000 | 1.000 |
| | **One regime with jumps** | **Two regimes with jumps (our model)** | | | | | | | |
| | | $q = 0$ | $q = 0.1$ | $q = 0.2$ | $q = 0.3$ | $q = 0.4$ | $q = 0.5$ | $q = 0.6$ | $q \geq 0.7$ |
| Risky-asset port | | | | | | | | | |
| Asia-Pacific | 0.505 | 0.353 | 0.431 | 0.482 | 0.531 | 0.594 | 0.448 | 0.007 | — |
| Europe | 0.167 | 0.157 | 0.151 | 0.159 | 0.198 | 0.367 | 0.552 | 0.993 | — |
| North America | 0.329 | 0.490 | 0.418 | 0.360 | 0.271 | 0.038 | 0.000 | 0.000 | — |
| Total risky wgt | 0.554 | 2.168 | 1.230 | 0.751 | 0.425 | 0.192 | 0.076 | 0.012 | 0.000 |
| Risk-free wgt | 0.446 | −1.168 | −0.230 | 0.249 | 0.575 | 0.808 | 0.924 | 0.988 | 1.000 |

This table provides optimal portfolio weights under four different models: one regime without jumps, one regime with jumps, two regimes without jumps, and two regimes with jumps. The investment universe consists of the ten country indices and the risk-free asset. The weights are presented by regions (Asia-Pacific, Europe, and North America). These regional weights are weights within the risky-asset portfolio and hence sum to one. The total weight for the risky-asset portfolio (Total risky wgt) and the risk-free weights (Risk-free wgt) are also provided. Means of normals and jumps are constrained to be equal across countries for each regime in the estimation. The estimates are based on the weekly data of the ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US) from January 2001 to December 2016. The investment horizon is one year, and the relative risk aversion coefficient is 5. The weights are provided for the probability of starting in the bad regime ($q$) of 0, 0.1, 0.2, ..., 0.6, and $\geq$ 0.7. Short selling is not allowed.

between countries. Of course, the difference in the country means is an important determinant of the portfolio weights in the real investment. So when we consider the predictive contribution for out-of-sample performance, we will estimate the model that allows the mean returns to vary across countries and regimes as detailed in the next section. Because most mutual funds are not allowed to short-sell shares, but are allowed to borrow (leverage) or invest in cash, we adopt this assumption in our analysis.[13]

Table 4 provides the optimal asset allocation weights which vary with the probability of the bad regime based on the full-sample data. For ease of discussion, we report the weights of the ten countries by grouping into three regions: Asia-Pacific (AU, HK, and JP), Europe (FR, GE, SP, SW, and UK), and North America (CA and US). These are regional weights within the equity portfolio and hence sum to one. These regional weights suggest how investors should diversify across regions, and the total risky and risk-free weights suggest how much leverage positions should be taken. To understand the improvement brought by our modeling, we also report portfolio weights of the models without jumps or with a single regime. Figure 4 provides a graphic illustration.

In our model, investors change their allocation based on the current probability of the bad market regime ($q$). Looking at our model, an investor, who is certain that the current regime is good ($q = 0$), will hold a leveraged position in equity. The equity allocation is 49% to America, 16% to Europe, and 35% to Asia-Pacific. That is not very different from market-capitalization weights. As $q$ increases, the investment in the risky assets drops and Asia-Pacific replaces America, while Europe is stable within the risky portfolio. Investors keep holding risky assets until $q = 0.6$. As the expected return decreases with the higher probability of a crisis, risk focus becomes more important. That is achieved both by a reduced leverage and a higher allocation to the region providing the best diversification benefits (lower correlation). But when the probability of a bad regime looms large (say $q$ over 0.4), the proportion of the risky assets gets smaller and a correlation break becomes more likely. This leads

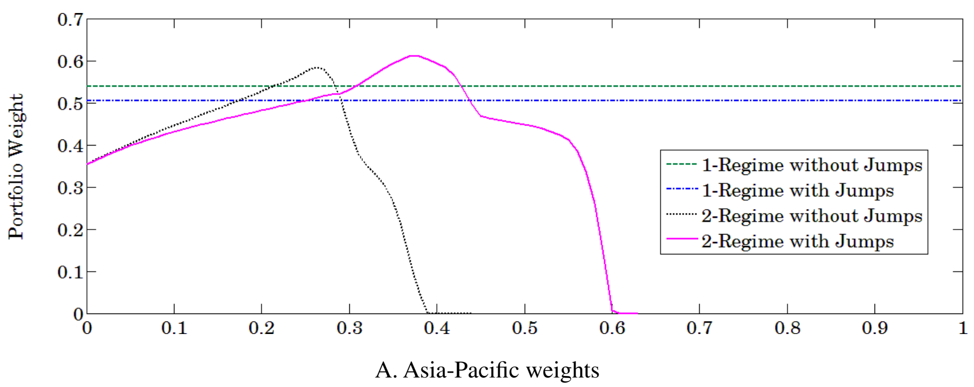



**Figure 4** Optimal portfolio weights.

This figure shows the optimal weights as functions of the bad regime probability of the four models: one-regime without jumps (dashed line), one-regime with jumps (dot-dashed line), two-regime without jumps (dotted line), and two-regime with jumps (solid line). The investment universe consists of the ten country indices and the risk-free asset. The weights in the Asia-Pacific, Europe, and North America regions are provided in Panels A, B, and C, respectively. They are the weights in the risky portfolio. The total risky weights are in Panel D. Means of normals and jumps are constrained to be equal across countries for each regime in the estimation. The estimates are based on the weekly data of the ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US) from January 2001 to December 2016. The investment horizon is one year, and the relative risk aversion coefficient is 5.

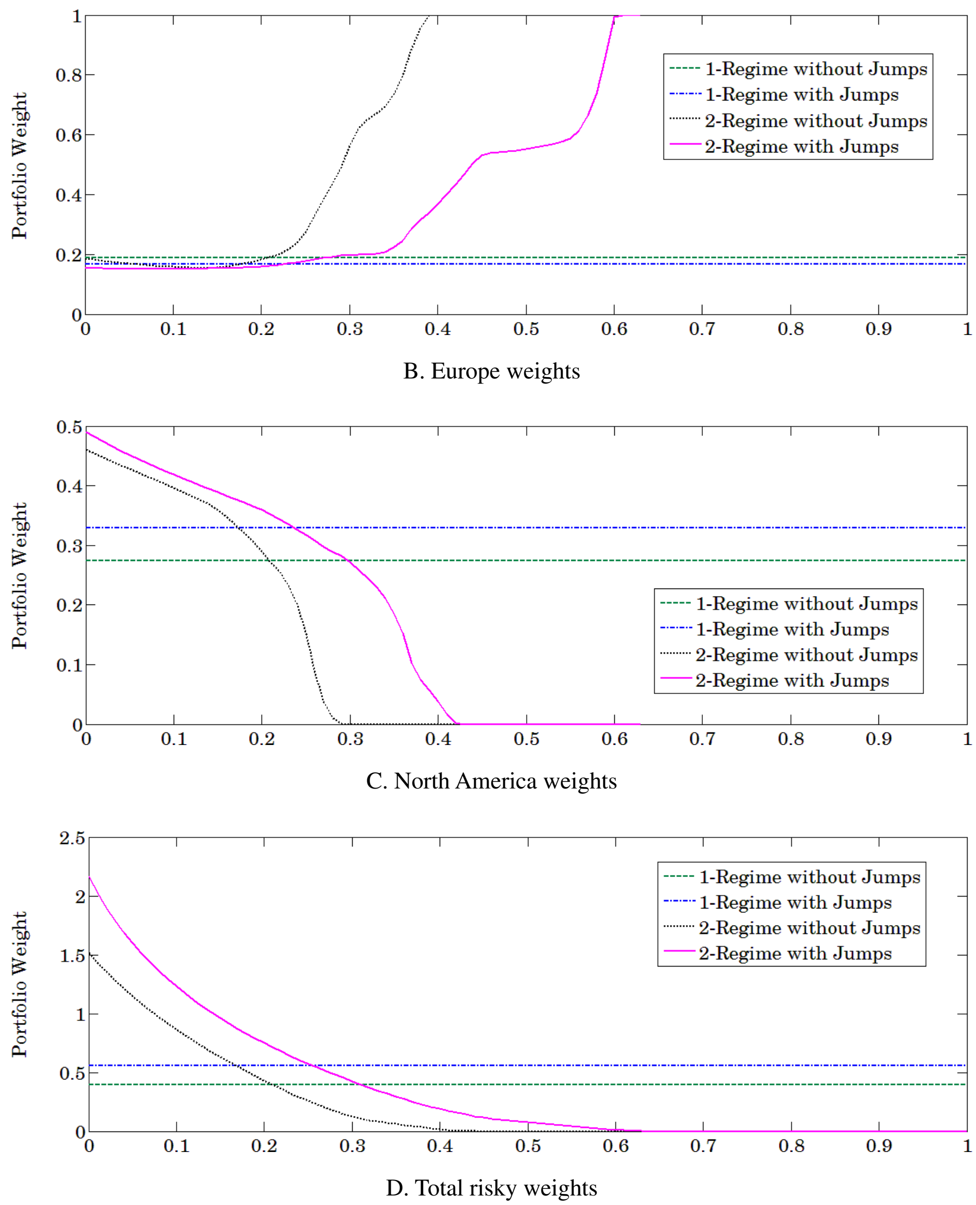



**Figure 4** (*Continued*)

to a drastic reduction in the Asia-Pacific allocation as the correlation break is most pronounced for this region. The allocation to Europe goes up as some European countries are less sensitive to correlation breaks, while the America allocation keeps dropping. When the probability $q$ goes over 0.6 there is no more allocation to equity.

In comparison, single-regime models, both without jumps (traditional MV[14]) and with jumps, are static with constant allocation. Including jumps in single-regime models induces a reduction in the weight of regions more sensitive to jump risk, such as Asia-Pacific. The North American weight is significantly increased relative to its MV weight.

Next, we compare the two-regime models without and with jumps. In the two-regime model without jumps, investors take less leverage for all probabilities $q$ and they stop investing in equity for $q$ around 0.4 (compared to 0.6 for our model). The equity allocations of both models are fairly similar when there is a large probability of a good regime (low $q$), but quite different when $q$ is above 0.2.

This suggests that correlation asymmetries between the good and bad regimes have substantial impacts on the composition within the equity portfolio. Improved risk modeling (including jumps) allows a better differentiation between the regimes. It also allows for taking more aggressive positions in risky assets for a similar perceived risk level and reflects changes in diversification benefits as the probability of a crisis evolves. To summarize, we get both leverage effects (adjusting the total risky weight) and portfolio composition effects (adjusting the country allocation).

To focus on the risk-modeling contribution, we imposed equal means. If we relax this restriction and use different means for different countries, even with shrinking them toward the equal-means as we do in Section 4, we get more extreme weights influenced both by expected returns and risk.

## 4 Predictive value: out-of-sample performance

The previous section was descriptive and we now turn to the predictive value of our risk model. A model with regime switching and jumps has a large number of parameters that may be subject to estimation errors. Testing the model out-of-sample is crucial. In a famous paper, DeMiguel *et al.* (2009) analyze out-of-sample portfolio performance of numerous mean–variance models, with and without taking into account estimation error, against the naive $1/N$ model.[15] They find that none of the models consistently outperforms the $1/N$ model and conclude that the gain from optimal diversification is more than offset by estimation error.

In this section we estimate the model allowing the countries to have different mean returns. As we all know, expected returns play an important role in asset allocation weights but using past data to predict returns is fraught with estimation error in the mean. To account for that, we use the shrinkage method of DeMiguel *et al.* (2013) in which the estimate of the mean return of each asset is shrunk toward the grand mean (average of the means across countries). In a Bayesian spirit, the country mean estimated from the data is adjusted toward a prior that all country expected returns are equal. Our analyses in this section rely on the shrinkage-mean model, but the results are qualitatively similar when means are not shrunk, or when means are constrained to be the same for all countries.

We now study the forward-looking results of the model against simpler models, including the robust $1/N$ model. Our out-of-sample period runs from January 2008 to December 2016, a total of nine years. This period covers various market crises (2008–2009, 2011, 2015–2016) and rallies (2009–2011, 2012–2014, 2016). We re-estimate the model every year using a seven-year rolling window. With the new estimates, we solve the dynamic portfolio optimization problem to obtain the optimal weights, which are functions of the market regime probability.[16] Table 5 provides the portfolio performance of five models: $1/N$, one regime without jumps, one regime with jumps, two regimes without jumps, and two regimes with jumps (our model). We report the annualized values of mean and standard deviation of excess returns (over the Fed Fund rate), the Sharpe ratio, and the maximum drawdown. We used a one-way transaction cost of 20 basis points (bp) for all trades. Our transactions are based on passive country indices for the ten largest

**Table 5** Out-of-sample portfolio performance.

| | Mean excess return (% per year) | Standard deviation (% per year) | Sharpe ratio | Maximum drawdown (%) |
|---|---|---|---|---|
| $1/N$ | 1.36 | 21.83 | 0.062 | 55.31 |
| One regime without jumps | −4.48 | 17.46 | −0.257 | 57.41 |
| One regime with jumps | −4.48 | 23.48 | −0.191 | 67.28 |
| Two regimes without jumps | 2.59 | 17.16 | 0.151 | 36.99 |
| Two regimes with jumps | 5.28 | 16.32 | 0.324 | 29.37 |
| MSCI World index | 3.36 | 19.69 | 0.171 | 54.70 |

This table provides out-of-sample portfolio performance of five different models: $1/N$, one regime without jumps, one regime with jumps, two regimes without jumps, and two regimes with jumps. It also provides the performance of the MSCI Word index. The results are based on the shrinkage-mean models of ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US). The entire sample data are from January 2001 to December 2016, and the out-of-sample data are from January 2008 to December 2016. The models are refitted at the beginning of each year. The trading frequency is weekly. The performances are measured by the annualized mean of excess returns, the annualized standard deviation of excess returns, the annualized Sharpe ratio, and the maximum drawdown. The one-way transaction cost of 20 bp is assumed. The risk aversion coefficient is 5, and no short selling is allowed.

equity markets, which would suggest lower costs. Back in 2010, Elkins/McSherry reported average overall transaction costs (commission, fee, and market impact) below 20 bp for the US and even lower costs for several other countries (i.e. Japan and France). With the growth in trading platforms and dark pools, total global transaction costs for institutional managers have gone further down. Some global institutional investors report average overall transaction costs around 10 bp.[17]

We find that our model with regime switching and jumps strongly outperforms the other four models in all aspects. Compared to the naïve $1/N$ strategy it provides a higher mean excess return (5.28% vs 1.36%) for a lower volatility (16.32% vs 21.83%). Hence its Sharpe ratio is much higher (0.324 vs 0.062). The single-regime models have negative returns. The model of two-regime without jumps is the second best but with a much lower excess return and Sharpe ratio. We also look at the passive MSCI World index made up of 23 developed markets as a proxy for market-capitalization-weighted index of our universe of the ten largest markets (well over 90% of the World index). The Sharpe ratio (0.171) is better than that of the $1/N$ strategy, but much lower than that of our model (0.324). As shown in the next figure, the return on MSCI World is better than the $1/N$ strategy because the US market (a big share of MSCI World) strongly outperformed most other markets in 2014–2016.

Figure 5 shows the accumulated wealth for all models. Our model shows a 67% increase in wealth while the two-regime model without jumps shows only an increase of 31%. The MSCI World shows an increase of 40% and the $1/N$ strategy shows an increase of 17%. On the loss side, our model has the lowest maximum drawdown of 29%, followed by the two-regime model without jump with the value of 37%. The other strategies have 55% or more maximum drawdown.

The strategy implies very frequent changes in asset allocation caused by any change in the regime probability. Hence the turnover and

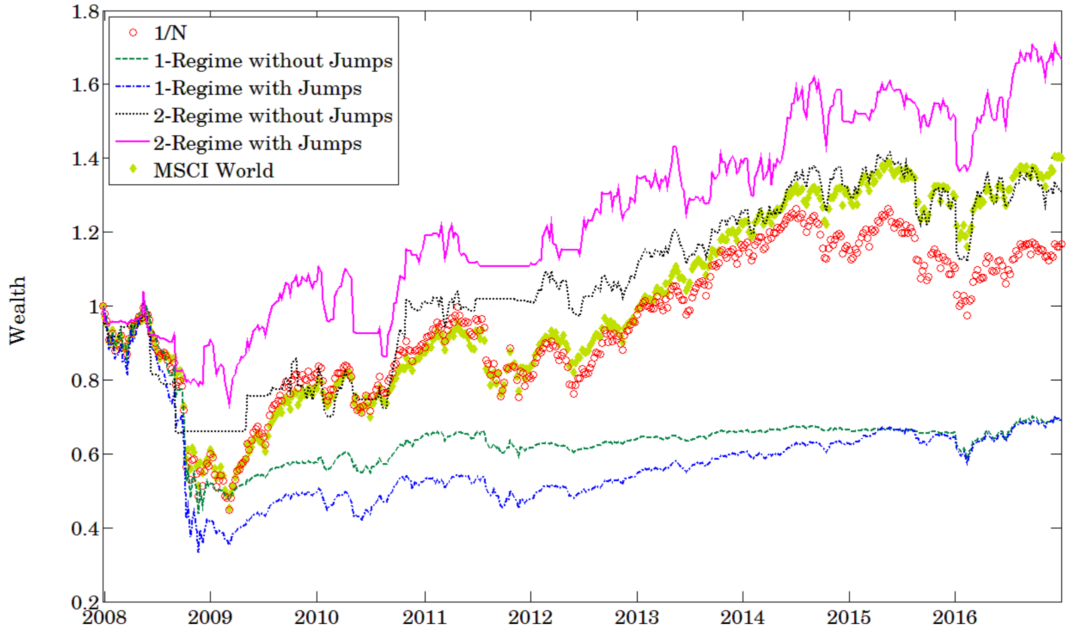



**Figure 5** Out-of-sample wealth processes.

This figure shows the out-of-sample wealth processes corresponding to five different trading models: $1/N$ (circles), one-regime model without jumps (dashed line), one-regime model with jumps (dot-dashed line), two-regime model without jumps (dotted line), and two-regime model with jumps (solid line). It also provides the MSCI World index normalized to one at the beginning of 2008. The results are based on the shrinkage-mean models of ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US). The entire sample data are from January 2001 to December 2016, and the out-of-sample data are from January 2008 to December 2016. The models are refitted at the beginning of each year. The trading frequency is weekly. The one-way transaction cost of 20 bp is assumed. The risk aversion coefficient is 5, and no short selling is allowed.

associated transaction costs are huge. We also test a more passive and realistic strategy. We keep the same allocation until there is a significant change in the regime probability (say more than 0.3 or 0.4) and then rebalance to the optimal weights given by the model. In other words, we only act when the probability of a bad regime significantly increases or decreases. Table 6 gives the results of that strategy for a threshold of 0.4 in the regime probability change.[18] The first six rows give the result of the strategy for increasing one-way transaction costs from 0 bp to 50 bp. The last row gives again the results for the MSCI World index. The results for the other models are vastly inferior and not reported here. Results for a transaction cost of 20 bp are better than before, as rebalancing is less frequent. Even with transaction costs of 50 bp, our model strongly dominates the passive World index in all respects (excess return, volatility, Sharpe ratio, and maximum drawdown). With a transaction cost of 50 bp, the final wealth shows an increase of 79% compared to 40% for the World index. There are 54 rebalancing over the 9-year period or an average of 6 per year. Trades are quite clustered: 50% of the trades are followed by another trade within the next 3 weeks. This emphasizes the benefit of using the weekly over monthly rebalancing schedule as a less-frequent rebalancing schedule could delay the trades and lead to poorer performance.

The reason for such a good performance is that we tend to detect crises very early. Regimes are persistent and the high correlation of observed jumps is a signal that a crisis has started. Hence the geographic portfolio composition is adjusted

**Table 6** Out-of-sample portfolio performance with infrequent rebalancing.

| Transaction cost (bp) | Mean excess return (% per year) | Standard deviation (% per year) | Sharpe ratio | Maximum drawdown (%) |
|---|---|---|---|---|
| 0 | 8.89 | 12.24 | 0.726 | 17.73 |
| 10 | 8.33 | 12.26 | 0.679 | 18.33 |
| 20 | 7.76 | 12.28 | 0.632 | 19.06 |
| 30 | 7.19 | 12.30 | 0.584 | 19.78 |
| 40 | 6.62 | 12.34 | 0.537 | 20.50 |
| 50 | 6.05 | 12.37 | 0.489 | 21.21 |
| MSCI World index | 3.36 | 19.69 | 0.171 | 54.70 |

This table provides out-of-sample portfolio performance of the model with two regimes and jumps with infrequent rebalancing strategy for one-way transaction costs of 0, 10, 20, 30, 40, and 50 bp. The trades occur only when the regime probability changes by 0.4 or more. It also provides the performance of the MSCI Word index. The results are based on the shrinkage-mean models of ten countries (AU, CA, FR, GE, HK, JP, SP, SW, UK, and US). The entire sample data are from January 2001 to December 2016, and the out-of-sample data are from January 2008 to December 2016. The models are refitted at the beginning of each year. The performances are measured by the annualized mean of excess returns, the annualized standard deviation of excess returns, the annualized Sharpe ratio, and the maximum drawdown. The risk aversion coefficient is 5, and no short selling is allowed.

to reflect the higher probability of a correlation break and leverage is reduced as well. We can see this clearly during the 2008–2009 and 2011 crises. On the other hand, a good regime is characterized by jumps of much smaller size and correlation. Shocks are easy to detect but it takes time to confirm that the process has become smoother. Our model takes time to estimate the return to a good regime.

It should be stressed that our approach is quite agnostic regarding expected returns. But returns tend to be low (negative) during crises, and breaks in correlation help us detect them quickly. It also helps to improve international diversification as the correlation structure evolves rapidly over time and crises are quite frequent. The geographic composition of a "defensive" portfolio when the probability of a bad market is high is quite different from that of an "aggressive" portfolio in good markets. Some serious caveats are in order here. The predictive value of better risk modeling shows promising potential, but our exercise is not actual implementation. There are other factors that could affect actual performance; for example, the difference in the borrowing and lending rates, and the lead time between updating regime probability and submitting orders. In practice asset allocation adjustments are likely to take a bit of time and therefore reduce the benefits of detecting crises early. The predictive performance also suffers a (small) bit of look-ahead bias; we know that our approach yields a good description of risk on the full sample. But our model is not based on maximizing return or performance.[19]

We do not use this excellent out-of-sample performance to suggest that one should take our model and implement it without further considerations. Rather, the focus of this section was to illustrate the benefits that could be gained from implementing better risk modeling on some predictive test covering nine recent years with many market cycles and crises.

## 5 Conclusion

All recent crises have led to a dramatic break in international correlation. Current risk management models often fail to depict it. The risk of a sudden break in correlation associated with higher volatility should be reflected in the optimal long-term asset allocation. Furthermore, market conditions (regimes) can change very quickly and are persistent; models of extreme correlation can detect such changes early and help adapt asset allocation tactically to changing market conditions.

In the past decades, we have experienced numerous and sudden global market crises. Unexpected shocks and sudden shifts in parameters have to be considered to improve risk management. In this paper, we focus on risk modeling and propose directions to improve risk management and asset allocation in global equity portfolios. We do not intend to propose trading rules based on return forecasts as our model is very agnostic in terms of expected returns. Results are mostly illustrative of improvements brought by better treatment of asset correlation and volatility. But expected returns do vary with volatility and correlation, and our approach enjoys this added benefit.

Clearly, asset prices do not follow simple unconditional normal distributions. Asset managers need more sophisticated risk management and asset allocation tools besides enhanced mean–variance or downside risk models that have been used in the past decades. A temptation is to use historical data to empirically fit a complex asymmetric distribution and numerically derive forward-looking asset allocation. A problem with these purely-empirical black boxes is that they provide little intuitive understanding of what is going on and how current market conditions drive changes in the recommended asset allocation. On the other hand a regime-switching model with jumps is an intuitive model based on distributions with well-known properties. In other words, it is sufficiently simple[20] and intuitive to be practically used by asset managers and integrated in the dynamic portfolio management process. The model is complex, but once programmed it will easily and automatically generate an updated regime probability which is central to rebalancing the asset allocation. As the reasons for changes in that probability are clear and intuitive, one could even incorporate subjective forward-looking beliefs of the asset manager in a Bayesian manner. This will provide better risk management and a direct method to improve risk-adjusted performance. It reduces the risk that global diversification fails at times it is needed the most.

## Appendix

### *A.1 Model*

Let $R_t = [R_{1,t}, \ldots, R_{n,t}]'$ denote the vector of returns of countries 1 through $n$ at time $t$. The return has the normal component and jump component:

$$R_t = Z_t + J_t, \quad t = 1, 2, \ldots$$

where $Z_t = [Z_{1,t}, \ldots, Z_{n,t}]'$ is the vector of normals, and $J_t = [J_{1,t}, \ldots, J_{n,t}]'$ is the vector of jumps at time $t$. There are two regimes: the bad regime (regime 1) and the good regime (regime 2). In the bad regime, the normal component of country $i$ or $Z_{i,t}$ is normally distributed with mean $\mu_i(1)$ and variance $\sigma^2_{Z,i}(1)$, and the correlation between $Z_{i,t}$ and $Z_{j,t}$ is $\rho_{Z,ij}(1)$. In the good regime, $Z_{i,t}$ is normally distributed with mean $\mu_i(2)$ and variance $\sigma^2_{Z,i}(2)$, and the correlation between $Z_{i,t}$ and $Z_{j,t}$ is $\rho_{Z,ij}(2)$ That is, the parameters of $Z_t$ in the bad and good regimes are different.

To describe jumps, we need two quantities. The first one is the number of jumps. Let $N_t$ denote the number of jumps in time period $t$, which is random. The mean number of jumps per period is $\lambda(1)$ in the bad regime and $\lambda(2)$ in the good regime. We assume that the number of jumps follows the Poisson distribution. So when the regime at time $t$ is $y$ (either 1 or 2), the probability of having $c$ jumps is

$$P(N_t = c \mid \text{regime is } y) = \frac{e^{-\lambda(y)}\lambda(y)^c}{c!}, \quad c = 0, 1, 2, \ldots$$

Recall that jumps are global or systemic. So if there is one jump in period $t$, every country has one jump in period $t$, and all countries "jump" at the same time.

The second quantity is the jump size. Let $\delta_{i,t}^m$ denote the jump size of jump $m$ of country $i$ in period $t$. When the regime at time $t$ is $y$, the jump size of each jump $m = 1, \ldots, N_t$ of country $i$ is normally distributed with mean $\eta_i(y)$ and variance $\sigma_{J,i}^2(y)$, and the correlation between $\delta_{i,t}^m$ and $\delta_{j,t}^m$ is $\rho_{J,ij}(y)$. Like the normals, the distributions of the number of jump and of jump sizes depend on the current regime. To summarize, we can write the return of country $i$ at time $t$ as

$$R_{i,t} = Z_{i,t} + \sum_{m=1}^{N_t} \delta_{i,t}^m.$$

The jump term (summation) is zero if there is no jump in period $t(N_t = 0)$. We assume that given the current regime, the normals and jumps are independent, and the jump sizes from different jumps (e.g. $\delta_{i,t}^m$ and $\delta_{i,t}^l$ for $m \neq l$) are independent.

Now we explain the regime-switching process in more detail. Let $Y_t$ denote the regime at time $t$, which could be 1 for the bad regime and 2 for the good regime. The regime is global. So if the regime is bad (good) at time $t$, it means all countries are in the bad (good) regime. The regime process follows the Markov chain process, which is described by a transition probability matrix

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where $p_{yk}$ is the probability that the next-period regime is $k$, given that the current-period regime is $y$:

$$p_{yk} = P(Y_{t+1} = k \mid Y_t = y).$$

For example, $p_{12}$ is the probability that the next-period regime is 2, given that the current-period regime is 1, while $p_{11}$ is the probability that the next-period regime is 1, given that the current-period regime is also 1. That is, $p_{11}$ is the probability that the regime remains the bad regime for one more period. Finally, the initial regime (regime at time $t = 1$) is random and has probability of $\varrho(1)$ that it is regime 1, and probability of $\varrho(2)$ that it is regime 2. In summary, the parameter set of our model is

$$\Theta = \{\varrho(1), \varrho(2), P, \mu(1), \mu(2), \sigma_Z^2(1), \sigma_Z^2(2), \rho_Z(1), \rho_Z(2), \eta(1), \eta(2), \sigma_J^2(1), \sigma_J^2(2), \rho_J(1), \rho_J(2)\}.$$

Our model can be viewed as a generalized discrete-time version of Das and Uppal (2004) who consider these types of systemic jumps. They, however, assume that the jump sizes $\delta_i$ are perfectly correlated across countries and consider a single regime. Our model also generalizes the model of Ang and Bekaert (2002) who consider a regime-switching model but without jumps.

### *A.2 Estimation*

One of Solnik and Watewai (2016) contributions is to derive an efficient estimation method based on the framework of the EM algorithm. Although tractable EM algorithms for certain regime-switching models have been proposed in the literature, deriving a tractable EM algorithm for multivariate models with regime-switching and jumps is nontrivial. Nevertheless, they are able to obtain a tractable algorithm for a large number of assets.

The EM algorithm is an iterative method for computing maximum likelihood estimators of model parameters when some variables are missing or unobservable. In our model, regime $Y_t$, normal $Z_t$, the number of jumps $N_t$, and jump size $\delta_t$ are unobservable. The algorithm has two steps, the expectation step (E-step) and the maximization step (M-step), which are performed alternately until convergence. More specifically, the algorithm starts with initial parameter values $\Theta^{(0)}$. Then we perform the first iteration of the E-step which computes a set of probabilities and expectations. We then use those quantities to compute the parameter values in the first iteration of the M-step to obtain $\Theta^{(1)}$. We alternate between E-step and M-step to obtain $\Theta^{(2)}, \Theta^{(3)}, \ldots$, until the parameter values do not change from one iteration to the next. Changing the initial $\Theta^{(0)}$ throughout the parameter space may increase the chance of getting to a global maximum solution. Further technical details can be found in Solnik and Watewai (2016). See also Dempster *et al.* (1977) for the general framework of the EM algorithm.

### *A.3 Regime probability computation*

Let $q_t$ denote the probability that the current regime is 1 (bad regime) given we have observed returns from time 1 to time $t$:

$$q_t = P(Y_t = 1 \mid R_1, \ldots, R_t).$$

Using the Bayes' rule, we can update the regime probability at time $t+1$ after observing the next return $R_{t+1}$ as follows

$$\begin{aligned} q_{t+1}(q_t, R_{t+1}) &= P(Y_{t+1} = 1 \mid R_1, \ldots, R_{t+1}) \\ &= \frac{[q_t p_{11} + (1-q_t)p_{21}] f_1(R_{t+1})}{[q_t p_{11} + (1-q_t)p_{21}] f_1(R_{t+1}) + [q_t p_{12} + (1-q_t)p_{22}] f_2(R_{t+1})} \end{aligned}$$

where $f_k(R)$ is the likelihood of return $R$ when the current regime is $k$:

$$\begin{aligned} f_k(R) &= \sum_{c=0}^{\infty} \frac{e^{-\frac{1}{2}(R-[\mu(k)+c\eta(k)])'(\Sigma(k)+c\Omega(k))^{-1}(R-[\mu(k)+c\eta(k)])}}{(2\pi)^{n/2}|\Sigma(k)+c\Omega(k)|^{1/2}} \\ &\quad \times \left(\frac{e^{-\lambda(k)}\lambda(k)^c}{c!}\right). \end{aligned}$$

This expression relies on the fact that the return is normally distributed when the number of jumps and regime are known. Note that $q_1 = f_1(R_1)\varrho(1)/\sum_{k=1}^{2} f_k(R_1)\varrho(k)$.

### *A.4 Portfolio weight computation*

We maximize the expected utility of the terminal wealth based on a power utility with relative risk aversion coefficient $\gamma$. Let $x_{i,t}$ denote the optimal portfolio weight of country $i$ at time $t$, and $r_f$ the risk-free rate of return. To compute the optimal portfolio weight for an investment horizon of $T$ time periods, we need to solve a dynamic programming problem.[21] We denote the *indirect utility* or *value function* at time $t$ when the probability of the bad regime is $q$ and the current wealth is 1 by $h(t,q)/(1-\gamma)$. This indirect utility measures the optimal expected utility as of time $t$. We need to solve the following recursive equation, known as the *Bellman equation*,

$$\begin{aligned} \frac{h(t,q)}{1-\gamma} = \max_x E\Bigg[&\frac{1}{1-\gamma} \\ &\times \left(e^{r_f} + \sum_{i=1}^{n} x_i(e^{\tilde{R}_{i,t+1}} - e^{r_f})\right)^{1-\gamma} \\ &\times h(t+1, q_{t+1}(q, \tilde{R}_{t+1})) | q_t = q\Bigg] \end{aligned}$$

backward in time from $t = T - 1, \ldots, 0$ with the terminal condition $h(T, q) = 1$ for all values of $q$. The Bellman equation states that the optimal expected utility at time $t$ comes from maximizing the utility of the portfolio return in period $t+1$, followed by optimizing the portfolio from time $t+1$ onward. The expectation in the Bellman equation is taken on $\tilde{R}_{t+1} = [\tilde{R}_{1,t+1}, \ldots, \tilde{R}_{n,t+1}]'$, which is the random vector of returns. The vector $x$ that maximizes the right-hand side of the Bellman equation is the optimal portfolio weight for time period $t+1$. See Solnik and Watewai (2016) for further technical details.

## Notes

[1] See, for example, Ang and Bekaert (2002), Hong *et al.* (2007), Okimoto (2008), and Chen (2016).

[2] For example, Engle (2002) proposes a multivariate GARCH model with time-varying correlations; Oh and Patton (2017) develop a factor copula model with asymmetric and fat-tailed factors to model dependence structure of stock returns; Omori and Ishihara (2012) develop a multivariate stochastic volatility model that allows asymmetries in correlations. As shown in Solnik and Watewai (2016), these models fail to match the correlations for extreme negative returns.

[3] See also Xiong and Idzorek (2012) for a comment on this issue.

[4] It can also depend on the investment horizon for horizon of one year or less. For on-going funds with a longer horizon, the allocation rules are approximately constant in the remaining investment horizon and depend solely on the regime probability.

[5] For more details about jump processes, we refer the reader to Cont and Tankov (2004).

[6] It should be noted that, in each time period, we observe only the return, which is composed of *either* normal alone *or* normal and jump. We rely on an estimation method to give us some probabilistic view about the possible values of normal and jump in the return in each time period. See the Appendix for the details.

[7] See Ang and Bekaert (2002), Honda (2003), and Guidolin and Timmermann (2007) for examples of asset allocation with regime switching. See Hamilton (2006, 2008) for more details about regime-switching processes.

[8] MSCI monthly total return data are available since 1970 but the data in early years are of lesser quality. For example, the total returns were adjusted for dividends by simply adding one-twelfth of yearly reported dividends. Many countries had severe restrictions on foreign investments, including capital and exchange controls that would make an active asset allocation strategy quite difficult.

[9] There are many parameters in our model, which are associated with simple distributions as described in the Introduction. The parameters are obtained from maximizing log-likelihood function using the expectation maximization (EM)-based algorithm. Detailed results are available from the authors.

[10] We tested whether a regime-switching model with jumps is a significant improvement over the models used in the past literature: (1) one regime without jumps (mean–variance), (2) one regime with jumps, (3) two regimes without jumps, and (4) two regimes with jumps (our model). We strongly reject simpler models. We tested if a third global regime was significant but found that it was not. We added country or regional jumps but failed to reject our model with only systemic jumps. Hence we opted for the more parsimonious specification with two regimes and systemic jumps. Note that models with more parameters may improve the likelihood value but are *not* necessarily preferred by the tests as they are "penalized" based on the number of parameters.

[11] Investors maximize expected utility of the terminal wealth and have a constant relative risk aversion $\gamma = 5$, a typical assumption. The equity portfolio composition is quite stable for different levels of risk aversion. This value of $\gamma$ generates strategies with volatility (standard deviation) being of an order of magnitude similar to that of passive global index strategies.

[12] The details of how to compute optimal weights and estimate the regime probability are given in the Appendix.

[13] The short-selling restriction rules out some hedge fund strategies arbitraging across markets, but these strategies are primarily based on country/market valuation (expected return), which is not our focus. We used the Federal Funds rate as lending/borrowing rate.

[14] Results are slightly different from the simple MV approach because we use expected utility maximization which also takes into account higher empirical moments.

[15] The $1/N$ model is an equally weighted portfolio in which each risky asset has the same portfolio weight and no weight is given to the risk-free asset.

[16] Remember that investors do not know with certainty what the current regime is, but estimate the probability $q$ of being in a bad regime.

[17] See for example, http://www2.westsussex.gov.uk/ds/cttee/pen/pen230714i4B.pdf.

[18] Results for a lower probability threshold of 0.3 are fairly similar.

[19] One could note, however, that the original model proposed by Solnik and Watewai (2016) used data up to early 2013. We added almost four years of data (196 weekly observations) and the out-of-sample performance on the extended period is good.

[20] Of course, the estimation method and the optimization algorithm are technically complex, but that is done by a computer.

[21] We refer the reader who is not familiar with dynamic programming to Puterman (2005).

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