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Applying regret theory to investment choices: Currency hedging decisions[☆]

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We apply regret theory, an axiomatic behavioral theory, to derive closed-form solutions to optimal currency hedging choices. Investors experience regret of not having chosen the ex post optimal hedging decision. Hence, investors anticipate their future experience of regret and incorporate it in their objective function. We derive a model of financial decision-making with two components of risk: traditional risk (volatility) and regret risk. We find results that are in sharp contrast with traditional expected utility, loss aversion, or disappointment aversion theories. We discuss the empirical implications of our model and its ability to explain observed hedging behavior.

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1. Introduction

"I should have computed the historical covariance of the asset classes and drawn an efficient frontier. Instead I visualized my grief if the stock market went way up and I wasn't in it – or if it went way down and I was completely in it. My intention was to minimize my future regret, so I split my [pension scheme] contributions 50/50 between bonds and equities." Harry Markowitz. As quoted in Zweig (1998), "America's top pension fund", Money, 27, p. 114.

Regret is a cognitively mediated emotion of pain and anger when agents observe that they took a bad decision in the past and could have taken one with better outcome. In financial markets, agents

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will experience regret when their investment yields, ex post, a lower performance than an obvious alternative investment they could have chosen. Contrary to disappointment, which is experienced when a negative outcome happens relative to prior expectations, regret is strongly associated with a feeling of responsibility for the choice that has been made. There is an extensive literature in experimental psychology and, to a lesser extent, neurobiology that supports the assumption that regret influences decision-making under uncertainty. Regret is such a powerful negative emotion that the prospect of its future experience may lead individuals to make seemingly sub-optimal, non-rational decisions relative to the expected utility paradigm. As the opening quote suggests, the anticipation of future regret was strong enough to turn Harry Markowitz away from his very own asset allocation theory when faced with a financial decision on his pension plan.

Bell (1982) and Loomes and Sugden (1982) derived an economic theory of regret. They propose a normative theory of choices under uncertainty that explains many observed violations of the axioms used to build the traditional expected utility (EU) approach. Regret theory (RT) assumes that agents are rational but base their decisions not only on expected payoffs ("value") but also on expected regret. Investors reach their investment decision by maximizing the expected value of this modified utility. So investors try to anticipate regret and take it into account in their investment decisions in a consistent manner. Risk takes two dimensions: traditional risk (volatility of final wealth) and regret risk. RT offers a parsimonious specification with strong cognitive and axiomatic foundations. It predicts Allais' paradox ("common consequences effect") and many other axiom violations[1] reported in experiments by Kahneman and Tversky (1979) and others.

Regret is clearly relevant to investment choices, when investors care about the outcome of their choice relative to other strategies they could have followed, passive benchmarks and peers. With the observed evidence in favor of the influence of regret on decision-making under uncertainty[2] as well as the axiomatic and normative appeal of RT for investment choices, it is surprising that RT has caught so little attention in the field of finance. Braun and Muermann (2004) apply RT to demand for insurance, and Muermann et al. (2006) apply RT to asset allocation in defined contribution pension schemes. Dodonova and Khoroshilov (2005) include some simplified form of regret in a model of asset pricing, but their utility function is not consistent with RT, as developed by Bell, Loomes and Sugden. All these models but focus on comparative statics. Comparative statics allow to get a sign on the influence of the level of regret aversion, but do not allow to derive explicit solutions for investment rules. Our methodological approach is quite different: using a second-order approximation, we can derive quantitative investment implications. Gollier and Salanié (2006) study the properties of a class of utility functions that exhibit some form of regret aversion, in an Arrow–Debreu economy. They derive some interesting implications for asset allocation decisions and asset pricing. In contrast to this existing literature, our methodology allows to derive closed-form (approximated) solutions for optimal investment choices. While the intuition of applying regret to currency hedging is not new (see Section 3), this is the first time that a formal theoretical approach is applied to currency hedging.

It must be stressed that RT, although intuitively appealing, is difficult to apply because of the technical difficulties associated with the optimization of an expected utility function with two attributes: value and regret. Indeed, applying RT to a general portfolio problem involving numerous assets is a difficult technical task. This is because regret stems from a comparison of the actual return outcome of each portfolio with the actual return outcome of all other feasible portfolios. This differs markedly from EU, where utility is solely defined over the chosen portfolio; hence portfolio risk is simply measured relative to preset expectations.[3] In addition, applying RT to dynamic decision-making where an agent makes decisions at different points in time renders the analysis even more intricate. These two

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[1] These are commonly referred to as the "common ratio effect", the "isolation effect", the "preference reversal effect", the "reflection effect", and "simultaneous gambling and insurance".

[2] Connolly and Zeelenberg (2002, p. 212) state that "the emotion that has received the most research attention from decision theorists is regret".

[3] The same comment applies to other behavioral extensions of traditional utility, where investors put a larger weight (or value) on losses than on gains relative to a reference point. Such a utility feature is often called "disappointment".

major technical difficulties probably explain the lack of applications of RT to the field of investment choices. For all those reasons, we deliberately apply RT to a financial decision that has been essentially studied within a simple static framework: the currency exposure decision. We are able to derive closed-form solutions and propose results that are amenable to economic interpretations and can be contrasted with those of the existing literature. Our model exhibits some form of loss aversion like other behavioral models but, more importantly, it also includes a form of risk that is very relevant to investment behavior. While currency exposure is the motivation of the model, the formulation is more general and can be applied to other problems of investment choices (e.g. bond–equity asset allocation), as discussed below.

We derive empirical implications that differ markedly from those of EU as well as from loss aversion models or disappointment aversion models. In the absence of currency-risk premium (zero expected return), regret-averse investors retain a currency-risk exposure, while all other theories suggest non-participation in risk exposure. Even with a zero-risk premium, regret-averse investors take a risk exposure for fear (potential regret) of missing a large gain on the risky asset. The currency case is interesting because a natural assumption for a currency-risk premium is zero due to the symmetric nature of an exchange rate between two currencies. In contrast, the equity risk premium is usually expected to be positive (although its actual magnitude is open to debate). Hence, the general empirical implication of regret theory is a positive currency exposure, while other theories imply zero exposure, i.e. full hedging. In the presence of positive (negative) currency-risk premium, regret-averse investors will increase (decrease) their currency exposure relative to the zero-risk premium case. However, the elasticity of exposure to the size of the risk premium is less than in EU, because regret aversion compounds risk aversion. Even in the presence of a large risk premium, regret-averse investors will be less inclined to take a full exposure to currency because that would create the potential for regret in case of large ex post currency depreciation. Finally, we find that regret-averse investors are highly sensitive to the skewness of the return distribution. We provide an empirical illustration based on a global survey of the hedging behavior of institutional investors.

To summarize, the first contribution of this paper is methodological; we incorporate regret in a tractable setting that allows us to derive explicit investment demand functions and nest traditional utility as a special case. Deriving explicit closed-form investment rules reflecting regret and risk aversions is novel. We show how investors should reflect their regret and risk aversions in optimal currency hedging. The optimal hedging rules can be readily applied to investment portfolios. Our second contribution is empirical; we derive empirical implications that are in marked contrast with EU theory and other behavioral models. Our regret model can be applied to a variety of corporate management decisions.

The paper is structured as follows. In Section 2, we introduce RT and compare it to other behavioral models. In Section 3, we discuss the importance of regret for currency decisions and describe our modeling of currency hedging. Section 4 derives closed-form hedging rules for currency-risk minimization (zero-risk premium), while Section 5 derives results for the general case with expectations on currency movements and correlation between asset returns and currency movements. Section 6 compares the predictions of our model to alternative decision theories and some observed hedging policies. Section 7 concludes this paper by outlining areas of application to management decisions.

2. Regret theory

As stated by Connolly and Zeelenberg (2002), regret is "the emotion that has received the most attention from decision theorists". There is an extensive literature in experimental psychology and a recent literature on neurobiology that shows that regret influences decision-making under uncertainty beyond disappointment and traditional uncertainty measures.[4] The psychology literature showed that the *experience* of regret is more intense when the unfavorable outcomes are the result of action rather than of inaction (Kahneman and Tversky, 1982) and that the *anticipation* of regret is

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[4] Experimental psychology reviews can be found in Gilovich and Medvec (1995) and Zeelenberg et al. (2000). Neurobiological experiments are discussed, e.g. in Camille et al. (2004).

taken into account in decision-making under uncertainty (Zeelenberg, 1999). Furthermore, the anticipation of regret is influenced by the visibility of the outcome of unchosen options (Zeelenberg, 1999). As to the neuroscience literature, Camille et al. (2004) showed that regret is neurologically different from disappointment and that the inability to experience regret distorts decision-making under uncertainty. Coricelli et al. (2005) measured brain activity during repeated lottery choices and showed that the experience of regret leads to regret aversion at the time of decision-making. There is also some evidence of cross-sectional differences in regret aversion across cultures or professional environments. Looking at the diagnosis of prostate cancer, Sorum et al. (2004) showed that US physicians are more averse to regret than their French counterparts. As this last reference shows, regret is an important emotion when analyzing real life decision-making in the fields of medicine (see also Smith, 1996), and it has also been studied in the context of consumer choices (e.g. Simonson, 1992; Cooke et al., 2001). In summary, regret seems to be an emotion that heavily influences the decision-making process and varies across individuals and environments.

Bell (1982, 1983) and Loomes and Sugden (1982) introduce regret in a theory of rational choice under uncertainty that is parsimonious yet can explain many of the observed axioms violations of EU theory. These authors derive a modified utility function of final wealth x resulting from a given investment choice a in a particular state of the world ω, knowing that a different investment choice b would have led to a final wealth y in the same state of the world:

$$u(x,y) = v(x) + f(v(x) - v(y)) \tag{1}$$

where $u(x,y)$ is the modified utility of achieving x, knowing that y could have been achieved. $v(x)$ is the traditional utility function, also called value function or choiceless utility. It is the "value" or utility that an investor would derive from outcome x if he experienced it without having to choose. This value function is assumed to be monotonically increasing and concave (risk aversion) as in traditional finance. The difference $v(x) - v(y)$ is the value loss/gain of having chosen a rather than a forgone choice b. $f(v(x) - v(y))$ indicates the regret of having chosen a, when b could have been chosen. The regret function $f(\cdot)$ is monotonically increasing and decreasingly concave,[5] with $f(0) = 0$. This modified utility $u(\cdot)$ is defined over the ex post (final) outcomes of investment choices; and rational investors would make choices ex ante by maximizing the expected value of this modified utility. Bell (1982, 1983) and Loomes and Sugden (1982) conclude that this is a well-behaved parsimonious functional form that allows to take regret into account in an axiomatic fashion and is consistent with empirically observed deviations from EU theory.

This functional form has been initially derived for pair-wise choices, but it can be extended (see Quiggin, 1994) to general choice sets. Consider that an investor can select among various investments i (e.g. some portfolio i), with outcome x_i. The modified utility of choosing investment i is given by

$$u(x_i) = v(x_i) + f(v(x_i) - v(\max[x_i])) \tag{2}$$

where $\max[x_i]$ is the best ex post outcome that can be obtained among all investments originally considered (feasible), the best forgone alternative. Note that the regret term $v(x_i) - v(\max[x_i])$ is always non-positive. Rational investors choose the optimal investment portfolio by maximizing their expected modified utility of all possible investment choices. So investors try to anticipate regret and take it into account in their investment decisions in a consistent manner.

It can be useful to highlight intuitively the difference with EU. As opposed to traditional von Neumann–Morgenstern utility, which is only defined over the actual portfolio owned by the agent, the modified utility also includes a comparison with other portfolios that could have been chosen but are not currently owned by the agent. Regret-averse investors do take into account traditional value, that is, they care about the expected return and volatility of their portfolio, but they also care about deviations from the (ex post) best forgone alternative. So there are two risk attributes in the utility function: volatility and regret risk.

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[5] Bell (1982, 1983) and Loomes and Sugden (1982) show that several behavioral patterns which contradict traditional expected utility theory are predicted by regret theory with a function $f(\cdot)$ that is concave for negative values of the argument and with $f''' > 0$, so that $f(\cdot)$ is decreasingly concave.

Other theories have proposed normative models of investment choices based on some preference-based characteristics that differ from EU. Two main classes of such models can be identified. *First*, there are models that rely on Prospect Theory (PT) (Kahneman and Tversky, 1979), a descriptive theory that has been extensively used in behavioral finance. Numerous authors have used PT in normative models of investment choices, i.e. maximizing some expected utility (e.g. Benartzi and Thaler, 1995; Barberis et al., 2001; Gomes, 2005). Utility models inspired by PT typically include a disappointment term with a kink at the current investment value (the "reference point") where the slope of utility is higher for losses than for gains ("loss aversion"). A *second* class of preference-based models (e.g. Ang et al., 2005) also assumes loss aversion, but derive these preferences from Disappointment Aversion (DA) theory (Gul, 1991), an axiomatic and normative decision theory using the Chew–Dekel class of risk preferences. Gul's preferences extend the expected utility framework by discriminating good and bad outcomes, i.e. outcomes above or below the certainty equivalent: bad outcomes are more heavily weighted than good outcomes. As a result, agents are more sensitive to bad outcomes and less to good ones. Ang et al. (2005) show that both LA and DA models imply first-order risk aversion[6]; in that sense LA and DA investors are "more" risk averse than EU investors. For a given lottery, agents with DA preferences will require a higher risk premium than an EU agent with the same utility function (typically a power utility).

Contrary to DA and LA, RT does not uniformly induce increased risk aversion relative to the expected utility paradigm. Our approach introduces two dimensions of risk. Loosely speaking, the first one is traditional volatility, linked to deviations of the chosen portfolio return from its expected value. The second one is regret risk, linked to deviations of the chosen portfolio returns from the return of the best forgone alternative. The two types of risks are neither identical nor fully correlated. Intuitively, regret induces a higher sensitivity to low-probability states with large payoffs. Compared to traditional investors who dislike volatility, regret-averse investors will bias their portfolios towards assets with high volatility, because these assets have a chance of a larger return relative to less volatile assets thereby creating the potential for large regret if they are not purchased. The skewness of the distribution affects regret. A positively skewed return distribution is such that there are low-probability states with very large returns. When return distributions are positively skewed, the anticipation of regret induces a higher exposure to risk than for symmetric distributions. We provide a more formal discussion in Section 5.

Another important difference between these theories is that DA and LA models can be incorporated in recursive dynamic settings using power utility functions, making it possible to analyze and compare their results with that of the traditional macroeconomics and finance literature. Incorporating RT preferences in a dynamic recursive model does not seem to be possible. Krähmer and Stone (2005) who study a two period dynamic regret model show that regret induces a backward-looking, path-dependent behavior. This feature would make a dynamic recursive regret theory model untractable. Hence, we simply consider a static one-period model.

The next step is to derive a model that incorporates regret aversion in financial decision-making.

3. Currency hedging: a regret-theoretic framework

We now briefly discuss why regret applies to the currency dimension and introduce our modeling framework.

3.1. Regret in currency exposure

Statman (2005) stresses that the currency exposure is a dimension where regret clearly applies (see also Gardner and Wuilloud, 1995). For example, an American investor who decided not to hedge

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[6] First-order risk aversion means that the risk premium required on a risky gamble is proportional to the volatility (σ) of the outcome. In traditional expected utility, we have second-order risk aversion, meaning that the risk premium is proportional to σ^2. In DA and LA models we have a kink in utility at the reference point. See also Backus et al. (2005).

currency risk would have incurred a currency loss of some 40% on its eurozone assets from late 1998 to late 2000, with a vast regret of not having fully hedged. Conversely a fully hedged investor would have missed the 50% appreciation of the euro from late 2001 to late 2004: again, a vast regret of not having taken the "right" hedging decision. The dollar exchange rate is widely publicized and deciding to invest part of a portfolio in a foreign currency, which can be equated to selling short the dollar, is an emotional decision where regret can easily be felt. The experience of regret in currency hedging is not news for the investment world. Numerous practitioners have justified a 50% naïve hedge ratio, the proportion of the foreign asset position that is hedged, on such intuitive grounds. For example:

> "The 50% hedge benchmark is gaining in popularity around the world as it offers specific benefits. It avoids the potential for large underperformance that is associated with "polar" benchmark, i.e. being fully unhedged when the Canadian dollar is strong or being fully hedged when it is weak. This minimizes the "regret" that comes with holding the wrong benchmark in the wrong conditions." Gregory Chrispin, "Managing Currency Risk: A Canadian Perspective", State Street Global Advisors, Essays and Presentations, March 23, 2004.

The 50% hedge ratio is the simplest currency hedging policy that attempts to deal with regret. This can easily be obtained by a simple minimax rule. We find that this behavior is optimal for one particular case, namely extreme regret aversion, and propose a more general theory for different levels of regret aversion.

We consider that the currency hedging decision is a residual one, once the global asset allocation has been chosen. This assumption is consistent with the traditional models of the hedging literature to which the main results of our model will be compared to. Furthermore, it is also consistent with the behavioral approach of mental accounting (or framing), the human tendency to treat each type of investment decisions in a separate mental compartment (see Tversky and Kahneman, 1981; Kahneman and Lovallo, 1993; Thaler, 1999). Rather than looking at the whole portfolio as prescribed by EU theory, investors tend to reach the best decision in each mental compartment. This feature is widely observed as far as the currency exposure decision is concerned. For example, such a behavior is confirmed in a survey[7] of Canadian pension plans. The vast majority of these plans (94%) believe that the best way to handle currency exposure is to decide first on global asset allocation and then handle the currency exposure. This confirms that the currency hedging decision is indeed taken as a residual/separate decision from the investment decision that creates the currency exposure. Accordingly, we do not claim to solve simultaneously the general problem of individual security selection, asset allocation between domestic and international assets and currency hedging. Although we believe there may be interesting interactions between portfolio choice and hedging decisions due to regret aversion, and that regret aversion may affect equilibrium asset prices, we do not touch upon these important issues in the present work. While currency hedging motivates our paper, we will illustrate how the proposed formulation also applies to other single-decision investment choices, such as the asset allocation decision between bonds and equities.

3.2. Modeling currency hedging

Before introducing our model, let us summarize the main currency hedging results of EU maximization. Solnik (1974), Adler and Dumas (1983) and Black (1990) derive an international asset pricing model where investors from different countries use their own currency as numeraire. Therefore investors from different countries view asset expected returns and risks differently because of foreign exchange uncertainty. Global equilibrium models conclude that all investors should hold a combination of their own risk-free asset (risk-free in home currency) and the world market portfolio partly hedged against currency risks. Hence the risky portfolio is identical for all investors and made of the equity market-capitalization-weighted portfolio optimally hedged against currency risk. A major result is that all investors should identically hedge their international investments, whatever their level of

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[7] William M. Mercer Investment Consulting's *Survey of Pension Plans On Currency Issues*, September 2000 (conducted in 2000 with responses from more than 100 large funds).

risk aversion. The risk level is adjusted by the combination of the risky portfolio and the national risk-free asset. The optimal hedge ratio is therefore "universal".

The traditional hedging literature determines the optimal hedge ratio, and hence currency exposure, by maximizing the investor's expected utility under the assumption that the asset allocation is already determined. If currency-risk premia are nil, a standard default assumption in foreign exchange markets, and if asset returns are uncorrelated with currency movements; then the optimal hedge ratio that maximizes expected utility is 100% because currency risk is pure noise (see Perold and Schulman, 1988; Anderson and Danthine, 1981). This means that investors with foreign invest-ments will not participate in currency exposure. Again, this hedge ratio obtains for any (positive) level of risk aversion or any portfolio composition and should therefore be "universal". This simple result obtains in the mean–variance case, as well as for any well-behaved utility function. Because difference in risk preferences is a major focus of our approach, we focus on this pure-risk case, but will also present results for more general return distributions.

We consider that the currency hedging decision is a residual one, once the global asset allocation has been chosen. Of their initial wealth, W_0, investors have allocated W_0^d to domestic asset and W_0^f to foreign assets $W_0 = W_0^d + W_0^f$. All valuations are conducted in domestic currency (e.g. the dollar for American investors). As in all currency hedging research, we do not focus on the interaction between domestic assets and foreign currency and will make the simplifying assumption, without loss of generality, that the final value of domestic assets, W^d, is non-stochastic. The dollar value of foreign assets is equal to the product of the foreign currency value of the foreign assets times the exchange rate (dollar value of foreign currency). Using log of price changes as return, the final (dollar) value of the foreign position, W^f, is $W^f = W_0^f(1 + \tilde{R} + \tilde{s})$, where $\tilde{R}$ is the return of the foreign asset in its local currency and s is the percentage currency movement (e.g. changes in the dollar value of the foreign currency).

Investors decide to hedge a proportion h of the foreign assets against currency risk, by selling the foreign currency forward. We assume that interest rates are equal worldwide, so that the forward exchange rate is equal to the spot exchange rate. Foreign assets are treated as a homogeneous asset class with a single currency. This is equivalent to saying that American investors care about an appreciation of the dollar against all currencies (a drop in the weighted average dollar value of foreign currencies, where the weights are those of the selected foreign asset allocation). A hedge ratio of zero implies no currency hedge and a hedge ratio of one implies full currency hedging. The participation in currency exposure is measured by $1 - h$. As forward contracts have a zero initial value, the initial wealth is unchanged by the hedging decision. Given a hedge ratio h, the final wealth value is given by $W = W^d + W_0^f(1 + \tilde{R} + \tilde{s}) - hW_0^f\tilde{s}$, so that

$$W = W^d + W_0^f\left(1 + \tilde{R} + \tilde{s}[1 - h]\right) = W^H + W_0^f\tilde{s}(1 - h) \tag{3}$$

where W^H refers to final wealth with full hedging.

For our purpose, the global asset allocation is fixed. Hence, the value (traditional utility) of final wealth, can be written as a function of h, the sole decision variable, and of the two stochastic variables $\tilde{R}$ and $\tilde{s}$, $v(\tilde{R} + \tilde{s}[1 - h])$. Note that derivatives satisfy the conditions $v' = W_0^f V'$ and $v'' = W_0^f W_0^f V''$. As the asset allocation is already fixed, we assume that investors exhibit regret on their only decision variable, the currency exposure. The modified utility can be written as

$$u\left(h, \tilde{R}, \tilde{s}\right) = v\left(\tilde{R} + [1 - h]\tilde{s}\right) + f\left(v\left(\tilde{R} + [1 - h]\tilde{s}\right) - v\left(\tilde{R} + \max\left[[1 - h]\tilde{s}\right]\right)\right) \tag{4}$$

where $v(\cdot)$ and $f(\cdot)$ are monotonically increasing and concave; $f(\cdot)$ is decreasingly concave ($f'' < 0$, $f''' > 0$) and $f(0) = 0$.

We impose the constraint $0 \leq h \leq 1$. A forward currency contract is equivalent to be long in one currency and short in the other. So a naked long position in the foreign currency is equivalent to a short position in the domestic currency. A hedge ratio greater than one (re-negative) would imply that the investor takes a naked short forward position in foreign (domestic) currency contracts that could lead to bankruptcy in extreme cases. If we assume that the value function satisfies the Inada condition (the first derivative tends to infinity when wealth tends to zero), the no-bankruptcy constraint should never be binding. So we limit the set of investments originally considered (feasible) by excluding speculative

positions in currencies. It must be stressed that, although at first sight, our results may appear to rely heavily on the assumptions made to the hedge ratio bounds ($h \in [0,1]$), we show in Appendix D that the results of our model remain qualitatively and quantitatively identical to the simple $h \in [0,1]$ case as long as short-sales constraints on foreign and domestic currencies are symmetric, even for arbitrarily large bounds. Implications of asymmetric short-sales constraints are also discussed in the Appendix.

With these assumptions, currency hedging is easy to analyze within a regret-theoretic approach because, with hindsight, the best forgone hedging decision can only be one of two possible choices that could have been made ex ante.

- *If the foreign currency appreciates* and whatever the positive value of *s*, the best forgone hedging alternative would have been to take the longest allowed position in the foreign currency. The best forgone alternative would have been to stay unhedged ($h = 0$). So for any positive *s*, $\max[[1 - h]s] = s$.
- *If the foreign currency depreciates* by any amount, the best hedging alternative would have been to take the shortest allowed position in the foreign currency. The best forgone alternative would have been to be fully hedged ($h = 1$). So for any negative *s*, $\max[[1 - h]s] = 0$.

Eq. (4) can thus be rewritten as

$$u\left(h, \tilde{R}, \tilde{s}\right) = v\left(\tilde{R} + [1 - h]\tilde{s}\right) + f_{s+}\left(v\left(\tilde{R} + [1 - h]\tilde{s}\right) - v\left(\tilde{R} + \tilde{s}\right)\right) + f_{s-}\left(v\left(\tilde{R} + [1 - h]\tilde{s}\right) - v\left(\tilde{R}\right)\right) \qquad (5)$$

Let us focus on the impact of a currency movement *s*. The utility $u(\cdot)$ is continuous and twice differentiable except in $s = 0$. At $s = 0$, the left-hand derivative with respect to *s* is equal to

$$\frac{\partial u}{\partial s} = (1 - h)v'\left(\tilde{R}\right) + (1 - h)f'(0)v'\left(\tilde{R}\right)$$

The right-hand derivative is equal to

$$\frac{\partial u}{\partial s} = (1 - h)v'\left(\tilde{R}\right) - hf'(0)v'\left(\tilde{R}\right)$$

At $s = 0$, the slope on the negative side is greater than on the positive side, as the difference $f'(0)v'(\tilde{R})$ is always positive. As a result, the utility function $u(\cdot)$ presents a kink at $s = 0$. Furthermore, the function $u(\cdot)$ is concave with respect to *s* (see Appendix A). The current exchange rate is a reference point and investors are more sensitive to reductions in financial wealth than to increases in financial wealth. These are common features in prospect theory. Here regret aversion induces currency "loss aversion", to coin a term frequently used in behavioral finance.

4. Derivations and results: currency-risk minimization

The optimal hedge ratio is obtained by maximizing the expected modified utility with respect to *h*. It can be noted that $u(h,R,s)$ in Eq. (5) is concave with respect to *h* (see Appendix B). To derive optimal investment rules, we need to make specific assumptions on the functions $v(\cdot)$ and $f(\cdot)$ to be used as well as on the distribution of *s*. If $f(\cdot)$ is linear (no regret aversion), then the problem reduces to EU maximization, as the maximization with respect to *h* of the expected utility given in Eq. (5) reduces to the maximization of $Ev(\tilde{R} + [1 - h]\tilde{s})$. In general $f(\cdot)$ is assumed concave (regret aversion). Except for very particular and simplistic functions $v(\cdot)$ and $f(\cdot)$, we cannot derive explicit hedging rules and would have to resort to numerical solutions with little generality.

The problem of deriving closed-form solutions already arises in the case of maximizing expected traditional utility, but the use of HARA utility functions and multivariate normally distributed assets' returns result in simple closed-form solutions. In our model, the problem is compounded by the presence of a piece-wise regret function defined over a value function. Because we want to obtain results that remain consistent with RT without losing the theoretical and empirical appeal of this approach, we use the two-moment approximation proposed by Pratt (1964) to conduct his analysis of risk aversion for small risks. We use a Taylor expansion of Eq. (5) and take its expected value, ignoring moments higher

than two. We then maximize with respect to h and are able to derive explicit hedging rules with interesting economic interpretation. This two-moment Arrow–Pratt approximation is very similar in spirit and results to the multivariate normality assumption for return distribution that was introduced in the finance literature. In both cases, we end up with models relying solely on the first two moments of return distributions. In traditional finance models, the normality assumption implies that for well-behaved utility functions, expected utility $\mathrm{Eu}(\cdot)$ can be expressed as a function of the means and covariances. In our model, the modified utility function is complex with two attributes, risk and regret. To allow for economic interpretation, we wish to explicitly retain the parameters of the modified utility in the optimal hedging rules derived from maximization of the expected modified utility. This cannot be done by simply assuming normality of returns. However, we can do it in the case of the Arrow–Pratt approach.[8] Most of the hedging literature has been using the two-moment assumption of multivariate normal distributions for $\tilde{R}$ and $\tilde{s}$, where the first two moments of the distributions are sufficient to characterize the whole distributions. As we will compare our results to this traditional mean–variance optimization, we are quite satisfied with making an equivalent two-moment assumption.

We start with the simple case of pure currency-risk minimization, where investors have no priors on expected currency returns or correlation between asset and currency returns. To make derivations even simpler, we assume that the distribution of currency returns is symmetric and that return on foreign assets is non-stochastic (without loss of generality, we set $R = 0$). We provide a discussion relaxing those assumptions in the next section.

The expectation of Eq. (5) under those assumptions can be written as

$$\mathrm{Eu} \ = \ Ev\Big([1-h]\tilde{s}\Big) + E_{s+}f\Big(v\big([1-h]\tilde{s}\big) - v\big(\tilde{s}\big)\Big) + E_{s-}f\Big(v\big([1-h]\tilde{s}\big) - v(0)\Big) \qquad (6)$$

where the notation E_{s+} denotes the expectation integral taken over positive values of s. As mentioned above, our problem is well-behaved as first derivatives of $u(\cdot)$ are well-defined and continuous, except in $s = 0$, and $u(\cdot)$ is concave in h and s. $\mathrm{Eu}(\cdot)$ is concave with respect to h as shown in Appendix B. The optimal hedge ratio satisfies the first-order condition: $(\partial \mathrm{Eu}/\partial h \ = \ 0)$. Because $\mathrm{Eu}(\cdot)$ is concave in h, this first-order condition is necessary and sufficient for optimality.

In the *absence of regret aversion* ($f(\cdot)$ is linear) the optimization problem reduces to EU optimization $\mathrm{Max}_h Ev([1-h]\tilde{s})$. Here $\tilde{s}$ is a pure risk (no expected return) and $1 - h$ is non-negative, therefore any risk-averse investor will attempt to eliminate that risk by setting h equal to 1. The participation in currency exposure $(1 - h)$ is therefore zero. This is the typical full hedging risk-minimization result. This leads to Proposition 1.

Proposition 1. *In the simple case with no currency-risk premium and non-stochastic asset return, the optimal hedge ratio for an EU investor is given by $h^* = 1$ for all levels of risk aversion.*

We now derive the analytical solution in the *presence of regret aversion* ($f'' < 0$) and obtain Proposition 2.

Proposition 2. *In the simple case with no currency-risk premium, symmetric currency return distribution and non-stochastic asset return, the optimal hedge ratio for a regret-averse investor is given by*

$$h^* \ = \ 1 - \frac{1}{2}\frac{v'^2 f''}{v''(1+f') + v'^2 f''} \ = \ 1 - \frac{1}{2}\frac{\rho}{\rho + \lambda} \qquad (7)$$

where $\lambda \ = \ -v''/v'$ is traditional risk aversion and $\rho \ = \ ((-v'f'')/(1+f'))$ is regret aversion.

Proof. See Appendix C. □

The optimal hedge ratio is equal to one (non-participation in currency exposure), as would obtain in risk minimization without regret, minus a term linked to regret aversion. Note that v' and f' are

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[8] Strictly speaking, the Arrow–Pratt approximation is valid for small risks. The quality of the two-moment approximation depends on the actual return distributions and the shape of the utility function. This has been extensively discussed in the literature, see Samuelson (1970), Loistl (1976), Levy and Markowitz (1979), and Kroll et al. (1984). We thank Christian Gollier for his support in getting a clearer view of this approach.

positive (investors prefer more wealth and less regret), v'' is negative (risk aversion) and f' is also negative (regret aversion), so the regret term is generally positive and lesser than one. We introduce the traditional Arrow–Pratt measure of local risk aversion $\lambda = -v''/v'$ and, following Bell (1983), define regret aversion ρ as $\rho = ((-v'f'')/(1+f'))$.

As mentioned previously, when the regret function $f(\cdot)$ is linear ($f'' = 0$), we are back to traditional utility maximization and an optimal hedge ratio of 1 (no participation in currency exposure). Since risk aversion and regret aversion are both positive, the optimal hedge ratio is always between 50% and 100%. Ceteris paribus, the lower the regret aversion ρ, the higher the optimal hedge ratio. When regret aversion ρ is very small relative to risk aversion λ (value dominates regret), h_* goes to 1 as ρ goes to 0. Conversely, when regret aversion is large relative to risk aversion, the optimal hedge ratio gets close to 50%. We call infinite regret aversion the case where regret aversion is very large relative to risk aversion. Note that this does not necessarily imply that ρ is infinite. It could also be that regret aversion dominates traditional risk aversion and that ρ is finite but λ is equal to zero. What really matters is the ratio ρ/λ. As shown in Appendix D, the results are unaffected by the constraints imposed on currency short-sales, as long as they are symmetric, i.e. as long the constraints are identical for short-selling the domestic or the foreign currency.

With this assumption of infinite regret aversion, our regret-theoretic model yields similar results to the "minimax regret" decision rule of Savage (1954). In this early model, Savage assumed that agents consider, for all possible decisions, the maximum regret that they may carry ex post. Agents then select the decision that carries the smaller such "maximum regret". Note that this decision is taken irrespective of the likelihood that such a regret may actually occur (provided that the probability is strictly positive). Intuitively, in our model with infinite regret aversion, investors care exclusively about the higher level of regret attained for any hedging decisions, whether in the region of gains or in the region of losses. When they hedge fully, investors anticipate that the maximum regret associated with a strong appreciation of the foreign currency, though unlikely, is so high that they reject such a hedging decision. Conversely, if they do not hedge at all, the regret associated with a strong depreciation in the foreign currency is again perceived as extremely high, even though it may be very unlikely, and is again rejected. As we assumed that the distribution of the foreign currency value is symmetric, the naïve 50% hedging policy will always be wrong and exhibit regret ex post. However, the maximum amount of regret will be cut in half whether it is attained in the region of gains or in the region of losses.

To summarize the case of pure currency-risk minimization, a regret-averse investor will always take some currency-risk exposure and hedge less than 100%, while an EU investor will never take a currency exposure, whatever her level of risk aversion. The optimal hedge ratio of 50% will only obtain for infinite regret aversion. In general the optimal hedge ratio will be between 100% and 50%, depending on regret aversion.

5. Derivations and results: general case

We now consider the general case where the return on foreign assets is stochastic and where the expected currency return can be non-zero. The expected value to be maximized with respect to h is

$$\text{Eu}\left(h,\tilde{R},\tilde{s}\right) = Ev\left(\tilde{R}+[1-h]\tilde{s}\right) + E_{s+}f\left(v\left(\tilde{R}+[1-h]\tilde{s}\right) - v\left(\tilde{R}+\tilde{s}\right)\right) + E_{s-}f\left(v\left(\tilde{R}+[1-h]\tilde{s}\right) - v\left(\tilde{R}\right)\right) \quad (8)$$

where $\tilde{R}$ and $\tilde{s}$ are stochastic with mean $\overline{R}$ and $\overline{s}$, so that $\tilde{R}=\overline{R}+\tilde{r}$, where $\tilde{r}$ is a random variable with zero mean, $\Sigma_s = E(\tilde{s}^2)$ and $\Sigma_r = E(\tilde{r}^2)$ and $\text{cov}(\tilde{r},\tilde{s})$ is the covariance between the two variables. The distribution is not necessarily symmetric. Note that Σ_s is not the variance but the expectation of squared values of s. Similarly we define Σ_{s+} as the (truncated) expectation of squared values of s taken over positive values of s: $\Sigma_{s+} := \int_0^{+\infty} s^2 g(s)\mathrm{d}s$ with $g(\cdot)$ the probability density function of s.

5.1. Traditional utility with no regret

To compare with the existing literature on hedging, let's first consider the special case where there is no regret. We obtain Proposition 3.

Proposition 3. *In the general case with a currency-risk premium, asymmetric currency return distribution and stochastic asset return, the optimal hedge ratio for an EU investor is given by*

$$h^* = 1 - \frac{\bar{s}}{\Sigma_s}\frac{1}{\lambda} + \frac{\mathrm{cov}(\tilde{r},\tilde{s})}{\Sigma_s} \tag{9}$$

where λ is traditional risk aversion.

Proof.   See Appendix E.   □

This result is the traditional one in the hedging literature. We will refer to it as the mean–variance case. It would be exact if the value function was quadratic or the distribution multivariate normal. Ceteris paribus, a positive expectation on the foreign currency movement $\bar{s}$ reduces the optimal hedge ratio (speculative term). The lesser the risk aversion, the lower the hedge ratio (investors speculate more). Similarly, a negative covariance between foreign asset return and currency movement (the local price of the foreign asset tends to go up when the foreign currency depreciates) reduces the optimal hedge ratio (covariance term). The term $(\mathrm{cov}(\tilde{r},\tilde{s})/\Sigma_s)$ can be thought as the elasticity, or beta, of asset returns to currency movements.

5.2. Modified utility with regret

The derivations for the general case in presence of regret are given in Appendix E. The optimal hedge ratio in the general case is given in Proposition 4.

Proposition 4. *In the general case with a currency-risk premium, asymmetric currency return distribution and stochastic asset return, the optimal hedge ratio for a regret-averse investor is given by*

$$h^* = 1 - \frac{\Sigma_{s+}}{\Sigma_s}\frac{\rho}{\rho+\lambda} - \frac{\bar{s}}{\Sigma_s}\frac{1}{\rho+\lambda} + \frac{\mathrm{cov}(r,s)}{\Sigma_s}\frac{\lambda}{\lambda+\rho} \tag{10}$$

where λ is traditional risk aversion and ρ is regret aversion.

Proof.   See Appendix E.   □

The hedge ratio is equal to 100% (risk aversion term) minus three terms:

$$\text{Regret term}: \quad h_{\mathrm{regret}} = -\frac{\Sigma_{s+}}{\Sigma_s}\frac{\rho}{\rho+\lambda} \tag{11}$$

This is similar to the expression $-(1/2)\,(\rho/(\rho+\lambda))$ of Proposition 2, derived with zero-risk premium and symmetric distribution, except that Σ_{s+} will generally differ from $(1/2)\Sigma_s$ if the expectation of s differs from zero or if the distribution is skewed. Hence, the previous discussion applies with one caveat. If $\bar{s} > 0$, Σ_{s+} will generally be greater than $(1/2)\Sigma_s$ (for a symmetric distribution) and investors will hedge less than in the risk-minimizing case because they anticipate to experience less regret if they decide not to hedge. Conversely they will hedge more if they anticipate the foreign currency to depreciate. A similar conclusion would obtain if the distribution is skewed. A positively skewed distribution implies that there are large currency returns with low probability. Regret-averse investors will bias their currency allocation towards these large, low-probability events to reduce the potential for regret. Therefore, they take more currency exposure (hedge less) than they would otherwise. Gollier and Salanié (2006) derive a similar result about the importance of skewness, with a different specification of regret in an Arrow–Debreu economy.

Note that investors with loss aversion or disappointment aversion are also particularly sensitive to skewness, but for a different reason. Because of the strong weight put on bad outcomes relative to good outcomes, disappointment-averse investors will dislike negative skewness and underinvest in risky assets (relative to the symmetric-distribution case). Regret-averse investors dislike positive skewness because of the potential for a large regret; to reduce regret risk, they overinvest in the risky asset (relative to the symmetric-distribution case). However, for both types of investors, negative (positive) skewness will lead to underinvesting (overinvesting) in the risky asset relative to the symmetric distribution case.

$$\text{Speculative term}: \quad h_{\text{specul}} = \frac{\overline{s}}{\Sigma_s}\frac{v'(1+f')}{v''(1+f')+v'^2f''} = -\frac{\overline{s}}{\Sigma_s}\frac{1}{\rho+\lambda} \tag{12}$$

As in the traditional mean–variance case (9), a positive expectation on the foreign currency movement reduces the optimal hedge ratio. The lesser the risk aversion λ, the lower the hedge ratio (investors take a larger currency exposure). But this is a modified risk aversion that takes regret into account. Regret aversion will, overall, add to traditional risk aversion: regret-averse investors tend to speculate less than traditional investors with the same level of risk aversion. Without concavity in the regret function f, the risk aversion would be similar to the traditional one. In general, regret adds to risk aversion because f'' is negative. Ceteris paribus, regret-averse investors will tend to "speculate" less on their anticipations of currency movements. However, as argued above, this effect will be mitigated by the regret term.

$$\text{Covariance term}: \quad h_{\text{cov}} = \frac{\text{cov}(\tilde{r},\tilde{s})}{\Sigma_s}\frac{v''(1+f')}{v''(1+f')+v'^2f''} = \frac{\text{cov}(\tilde{r},\tilde{s})}{\Sigma_s}\frac{\lambda}{\lambda+\rho} \tag{13}$$

In the absence of regret aversion $\lambda/(\lambda+\rho) = 1$ and the covariance hedging term is identical to that in the traditional mean–variance case as shown in (9). However, in the presence of regret, $\lambda/(\lambda+\rho)$ is less than one and investors tend to deviate less from their risk-minimizing hedging policy than traditional investors. The intuitive explanation is straightforward. A negative correlation between foreign asset return and currency movement implies that asset returns tend to soften the impact of currency risk at the portfolio level; but regret is only measured on the currency movement itself, not on asset return. The value function in the modified utility takes into account total portfolio risk and suggests a lower hedge ratio because of the negative correlation, but this is partly dampened by regret aversion on currency losses.

Note that with infinite regret aversion, the speculative and covariance terms are equal to zero, and the optimal hedging policy is only influenced by the regret term. In this case, the optimal hedge ratio will be equal to 50% when Σ_{s+} is equal to $(1/2)\Sigma_s$, as in the risk-minimizing case. However, regret aversion may push the optimal hedge ratio lower (higher) than 50% if Σ_{s+} is more (less) than $(1/2)\Sigma_s$. This could be the case when the expected currency movement is positive (negative) and/or when the distribution is positively (negatively) skewed.

5.3. The interaction between risk and regret aversions

We now discuss how the relative levels of regret and risk aversions affect hedging policies. We provide in Fig. 1 the optimal hedge ratio and its three components (regret term, speculative term and covariance term) for different levels of regret aversion. We arbitrarily set risk aversion at one, and let regret aversion vary from 0 to 15. As an illustration we use different assumptions on market expectations. In all cases, the standard deviation of percentage currency movements is set at 10% per year (so $\text{var}(s) = 1\%$). Fig. 1 presents the case where investors hold no expectations about currency movements so that $\overline{s} = 0$ and $\text{cov}(\tilde{r},\tilde{s}) = 0$. As discussed previously, we find that the risk-minimizing hedge ratio is 100% in the absence of regret aversion. The optimal hedge ratio reaches 75% when regret aversion equals risk aversion and drops to 50% when regret aversion dominates risk aversion.

Fig. 2 introduces a positive expectation on the foreign currency with $\overline{s} = 1\%$ per year. In the absence of regret, the optimal hedge ratio is 0%, as investors choose a full currency exposure[9] (speculative term exactly offsets risk aversion term). As regret aversion increases, the optimal hedge ratio increases. This is caused by a strong increase in the speculative term in Eq. (12) as ρ increases: remaining unhedged creates the potential for a strong regret that is not offset by the utility of the higher expected return. The optimal hedge ratio is 25% when regret aversion equals risk aversion, and reaches 50% when regret aversion dominates risk aversion.

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[9] The ratio of expected currency movement to variance is equal to one when $\overline{s} = \Sigma_s = 1\%$. For a risk aversion of one, this translates into a speculative term of -100% for the hedge ratio.
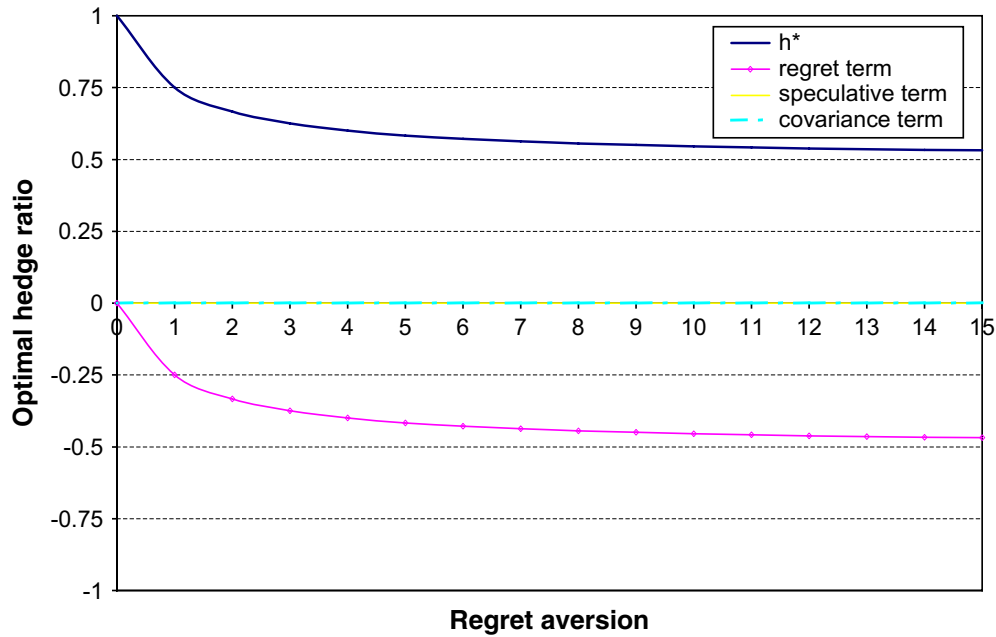
Fig. 1. Optimal hedge ratio, $\bar{s} = 0$ and $\text{cov}(\tilde{r}, \tilde{s}) = 0$.

Fig. 3 introduces a positive covariance between asset return and currency with a positive expectation on the foreign currency. In other words, the price of the foreign asset, measured in foreign currency, tends to drop when the foreign currency depreciates. This is an unpleasant feature from a risk viewpoint as it increases the impact of a currency loss. It leads to a covariance term that increases the amount of optimal hedging. In the simulation, the expected currency appreciation is 1% and the elasticity of asset return to currency movement is 0.2. In the absence of regret, investors would adopt a hedge ratio of 20% (the speculative term is −100% and the covariance term is 20%). A regret-averse investor with a regret aversion similar to risk aversion would only hedge partially (34%); but the optimal hedge ratio would reach 50% when regret aversion dominates risk aversion.

In general, these results suggest that regret aversion may increase or decrease the optimal exposure to risk relative to EU investors. When a traditional investor takes little or no exposure to currency risk
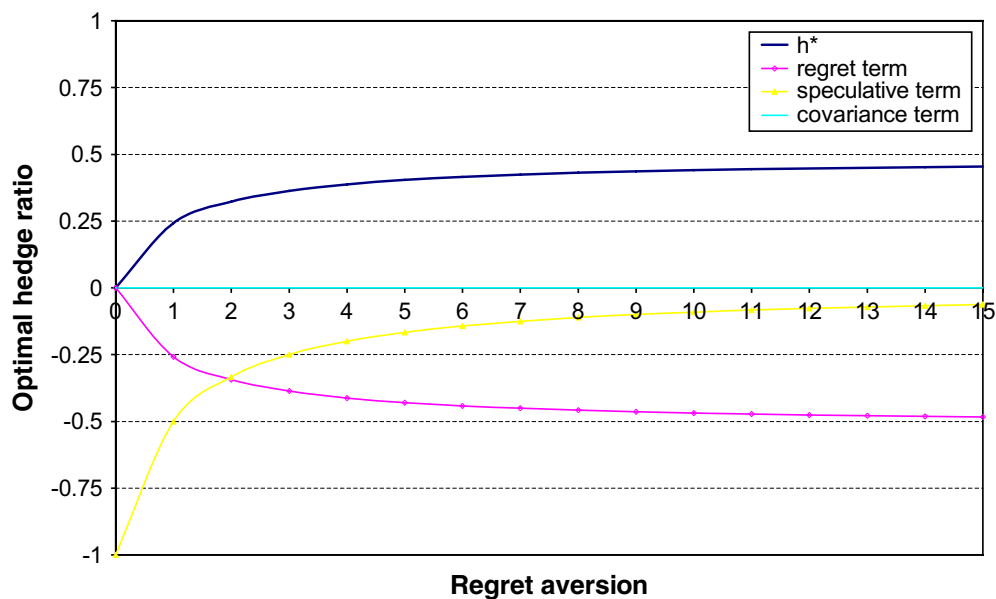

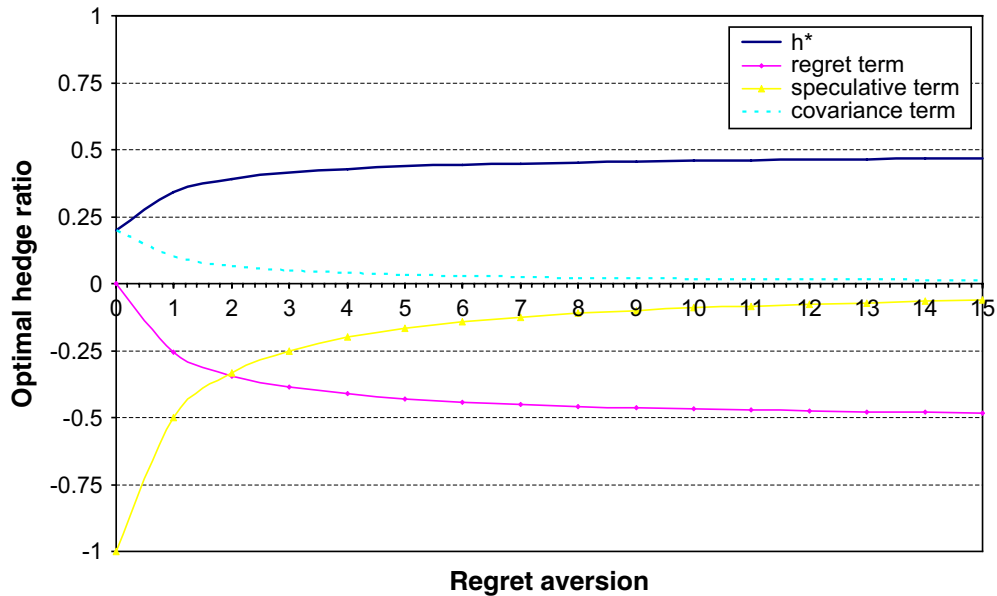**Fig. 2.** Optimal hedge ratio, $\bar{s} = 1\%$ and $\text{cov}(\tilde{r}, \tilde{s}) = 0$.

Fig. 3. Optimal hedge ratio, $\bar{s} = 1\%$ and $\mathrm{cov}(\tilde{r}, \tilde{s})/\Sigma_s = 0.2$.

(Fig. 1), a regret-averse investor tends to take a higher exposure, for fear of missing upon currency appreciation. On the other hand, when a traditional risk investor exposes to currency risk for speculation motives (Figs. 2 and 3), a regret-averse investor will take a lower exposure because regret aversion adds to risk aversion. While the simulations have been conducted for agents with different levels of regret aversion but a fixed level of risk aversion, it could be misleading to consider regret and risk aversions independently. The two attributes are likely to have some degree of substitutability. For example, an agent with high regret aversion could have lower risk aversion and vice versa.

Looking at these results, a natural question arises as to whether regret aversion can be observationally equivalent to risk aversion. In other words, can a risk-averse investor ($\rho = 0$ and a risk aversion λ_0) exhibit the same hedging behavior as a regret-averse investor (with some ρ and λ)? Because ρ and λ enter the three hedging terms in a different fashion ($\rho/(\rho + \lambda)$, $\lambda/(\lambda + \rho)$ and $1/(\rho + \lambda)$), a traditional investor with no regret aversion would not adopt the same hedging rule as a regret-averse investor. In other words, we cannot find a local risk aversion λ_0 that would yield the optimal hedging rule described above: for each set of market expectations (currency expected return and variance) we would get a different value of λ_0. But local risk aversion is a property of the utility function defined over current wealth, it is not supposed to be dependent on the parameters of the distribution of future returns. One can be tempted to define "overall risk aversion" as the sum of regret aversion and traditional risk aversion ($\rho + \lambda$). But hedging behavior will be different depending on the two components of overall risk aversion.

Our results could provide a framework to estimate agents' regret and risk aversions. We could think of a laboratory experiment where we give investors different scenarios about expected currency movement and elasticity of asset return to currency movement. As ρ and λ enter the three hedging terms in a different fashion, we can derive estimates of both regret aversion and risk aversion.

6. Empirical implications

We now discuss the empirical implications of our model and its ability to explain some of the observed hedging behavior relative to other models of decision theory. Our model is normative and does not derive equilibrium pricing implications. Hence it cannot be tested using market data; the same caveat applies to most of the hedging literature. What we do here is compare the empirical implications of our model and those of the other hedging models. We also look at some survey data of institutional investors regarding their currency hedging policy.

6.1. Comparison with other theories

We primarily focus our discussion on risk-minimization hedging behavior. As opposed to other risky assets such as equity, it is unclear whether a given currency should trade at a positive or negative risk premium relative to other currencies. So the case of a zero currency-risk premium is an important one. Furthermore, the risk-minimization case provides the sharpest difference in empirical implications between our model and other approaches.

A major result of EU is that investors do not take a currency exposure in the absence of a currency-risk premium and priors on correlation between asset and currency returns, whatever their level of risk aversion. Here, we find a markedly different result: regret-averse investors would take a currency exposure (hedge ratio less than 1) in a pure risk-minimizing strategy. Regret-averse investors tend to retain foreign currency participation, despite the absence of risk premium, in order to reduce their regret in case the foreign currency appreciates. A fully hedged portfolio (non-participation in currency exposure) leads to the potential of large regret risk. This empirical implication is also at odd with other behavioral models such as loss aversion models inspired by prospect theory or disappointment aversion models inspired by the Chew–Dekel class of risk preferences. As discussed in Section 2, LA or DA investors exhibit higher risk aversion than EU maximizers as they exhibit first-order risk aversion. So, if anything, that would reinforce the non-participation effect. The empirical implication of all these alternative behavioral models is that we should observe a tightening around 100% of the dispersion of hedge ratios empirically observed among investors.

However, in the presence of currency-risk premia, regret-averse investors will tend to speculate less than traditional investors with similar risk aversion. The elasticity of exposure to the size of the risk premium is less than in EU, because regret aversion compounds risk aversion.

Currency hedging has motivated our model of optimal portfolio choices, but the same formulation can readily be applied to other investment problems. For example, we could apply our formulation to the allocation between a risk-free asset and a risky asset called equity. In our notations, R would be the return on the risk-free asset, $\tilde{s}$ the excess return on equity and h the proportion invested in the risk-free asset ($1 - h$ invested in equity). Even with a zero equity risk premium, regret-averse investors would participate in the equity market to avoid regret of missing a large equity return. This is in sharp contrast with EU or LA and DA models.[10] However, the equity market participation of regret-averse investors is less sensitive to the magnitude of the risk premium than for EU investors. Regret-averse investors will increase their equity participation if the equity risk premium increases, but regret aversion compounds risk aversion thereby reducing the elasticity to the magnitude of the risk premium. A full market participation runs the risk of experiencing strong regret in case of an unexpected drop in equity price. Such an intuition is also implicit in Muermann et al. (2006) and Gollier and Salanié (2006).

Regret theory concludes that the skewness of the distribution strongly affects investment choices, relative to traditional utility. Positive skewness induces investors to increase their participation in the risky asset. This is also an empirical implication of LA and DA models.

6.2. Survey of institutional investors

Empirical studies of the actual currency hedging policy adopted by investors are scarce and address the foreign currency exposure of corporations,[11] not portfolio investments. However, an interesting global survey (Harris, 2004) of institutional investors conducted by Mellon/Russel can be used to illustrate our conclusions. Fig. 4 gives the distribution of currency-hedge ratios for 563 institutional investors delegating the currency hedging decision to overlay managers. Each column gives the distribution of the hedge ratio for investors from a given base currency, i.e. a region (e.g. the first column is the distribution for US investors), as well as the number of accounts in that base currency. The last

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[10] Ang et al. (2005) also show that non-participation obtains in a DA model even in the presence of a positive risk premium if disappointment aversion is sufficiently large (p. 476).

[11] For example, The 1998 Wharton survey of financial risk management by US non-financial firms questioned firms on their hedging policy for various foreign transactions and balance sheet exposures. It appears that the average hedge ratio for on-balance sheet items is 49%; see Bodnar et al. (1998).
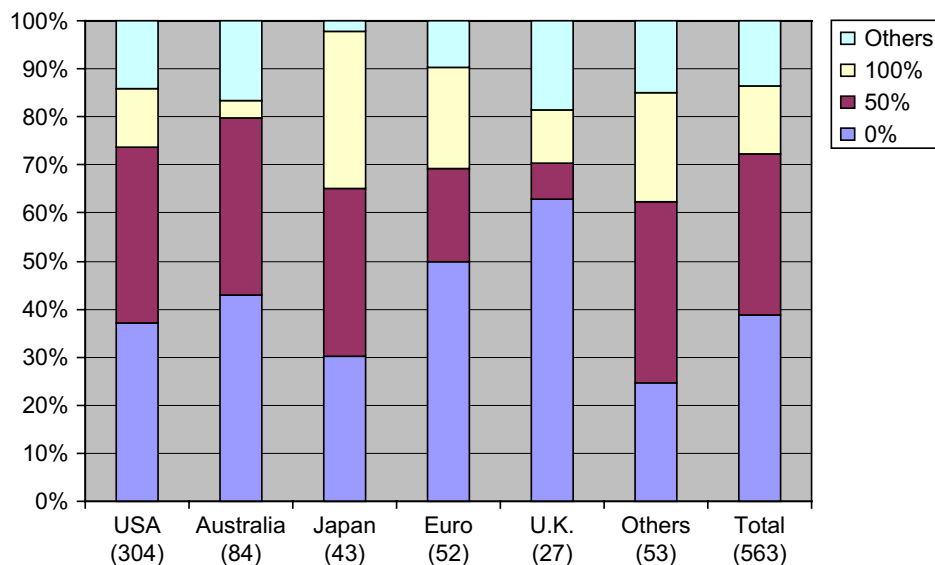
Fig. 4. Distribution of hedge ratio for investors from several base currencies.

column gives the distribution of all accounts. The hedge ratio is actually the benchmark assigned to currency overlay managers, not the actual hedge ratio actually used. We are well aware that this can create all kind of potential biases. Unfortunately, this is the only extensive survey that we could find. And it is only used to illustrate our discussion of empirical implications.

Looking at the worldwide average, 39% of investors adopt a no-hedging policy, 34% of investors adopt a 50% hedging policy, 14% of investors adopt a 100% hedging policy and 13% of investors adopt some other hedge ratio. In the cross-section, US, Australian, and Japanese investors favor a 50% hedging policy much more than their British and Euroland counterparts. Because the numbers presented in Fig. 4 reflect long-term hedging policy (benchmark), they cannot be explained by short-term expectations on currency movements, but primarily by risk considerations or some behavioral attitude. Remember that EU models, as well as alternative behavioral models, predict a 100% hedge ratio, i.e. no currency participation, in the absence of a risk premium. This prediction seems at odd with some of the evidence presented in Fig. 4. A significant proportion of investors adopt a 50% hedging policy. They could be regarded as investors with large regret aversion. The observation that many more investors use a 50% than a 100% hedging policy could suggest that regret aversion is more relevant than risk or disappointment aversion for currency hedging decisions; but the data are too partial and potentially biased to make this statement reliable.

So far, we discussed the dispersion of hedge ratios across all investors. It is also interesting to consider dispersion across nationalities. Assuming that regret aversion is what drives hedging behavior, the data reported in Fig. 4 would suggest that American investors are more regret averse than European investors: 37% of Americans favor a 50% hedging policy, while only 7% of British and 20% of Eurolanders do so. Cultural and environmental differences could explain national differences in regret aversion. For example, American institutional investors focus much more on relative performance than their European counterpart. One could argue that underperforming peers (i.e. other investment policies that could have been chosen ex ante) is more painful in America than in Europe. But there can be another explanation consistent with traditional EU if asset and currency returns are correlated. Hau and Rey (2006) and Campbell et al. (2007) report that the correlation between asset and currency returns differs by country and could justify different levels of hedging for investors of different nationalities.

7. Conclusion

We studied a model of optimal currency hedging choices based on regret theory and derived (approximated) closed-form solutions. There are two risk attributes entering the objective function,

traditional volatility and regret risk. Regret for having taken a wrong decision seems an important psychological trait in investment choices. It is widely apparent in the selection of a foreign currency exposure on portfolio investments. Regret is experienced if the outcome of unchosen options is "visible", and currency returns are highly visible and emotional. Everyone, even outside the sphere of finance, seems to have an opinion on the value of the dollar, especially ex post. In addition, all performance reports separate the currency gains/losses on the portfolio from other sources of return. Performance relative to peers or other simple hedging strategies are important. In institutional asset management, what might have been a reasonable risk-averse hedging decision ex ante, can be easily criticized ex post by a board of trustees.[12] Here we provide hedging rules that take into account both risk and regret aversions and should therefore be useful to the asset management industry.

We mentioned that our model can be extended to other portfolio investment decisions, such as the asset allocation between bonds and stocks. It could also be applied to other corporate decisions. For example, airlines engaged in policies of hedging oil price risk when oil prices moved above $70 a barrel in 2006. This policy subsequently created huge cash losses, and vast regret, when oil prices dropped to $60. Multinational firms engage in corporate hedging of currency exposure on their cash flows and balance sheet. Regret is felt when an asset (e.g. a foreign subsidiary value) is hedged against the depreciation risk of the foreign currency which turns out to appreciate ex post. Regret can be made more visible by accounting rules that might not allow the writing up of the book gain in asset value, while the loss on the hedging liability, sometimes in the form of cash if derivatives are used, is immediately recognized.

Acknowledgements

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jimonfin.2008.03.001.

References

Adler, M., Dumas, B., 1983. International portfolio choice and corporate finance: a synthesis. Journal of Finance 38, 925–984.
Anderson, R.W., Danthine, J.-P., 1981. Cross hedging. Journal of Political Economy 89, 1182–1196.
Ang, A., Bekaert, G., Liu, J., 2005. Why stocks may disappoint. Journal of Financial Economics 76, 471–508.
Backus, D.K., Routledge, B.R., Zin, S.E., 2005. Exotic Preferences for Macroeconomists. In: Gertler, M., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2004, The MIT Press.
Barberis, N., Huang, M., Santos, T., 2001. Prospect theory and asset prices. Quarterly Journal of Economics 116, 1–53.
Bell, D.E., 1982. Regret in decision making under uncertainty. Operations Research 30, 961–981.
Bell, D.E., 1983. Risk premiums for decision regret. Management Science 29, 1156–1166.
Benartzi, S., Thaler, R., 1995. Myopic loss aversion and the equity premium puzzle. Quarterly Journal of Economics 110, 73–92.
Black, F., 1990. Equilibrium exchange rate hedging. Journal of Finance 45, 899–907.
Bodnar, G., Hayt, G., Marston, R., 1998. 1998 Wharton survey of financial risk management by US non-financial firms. Financial Management 27, 70–91.
Braun, M., Muermann, A., 2004. The impact of regret on the demand for insurance. Journal of Risk and Insurance 71, 737–767.
Camille, N., Coricelli, G., Sallet, J., Pradat-Diehl, P., Duhamel, J.R., Sirigu, A., 2004. The involvement of the orbitofrontal cortex in the experience of regret. Science 304, 1167–1170.
Campbell, J.Y., Serfaty-de Medeiros K., Viceira L., 2007. Global Currency Hedging, Unpublished working paper.

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[12] Furthermore, selling short an appreciating foreign currency leads to cash losses on the forward position that have to be covered by the sale of assets. A forced decision that is painful.

Connolly, T., Zeelenberg, M., 2002. Regret in decision making. Current Directions in Psychological Science 11, 212–216.

Cooke, A.D., Meyvis, T., Schwartz, A., 2001. Avoiding future regret in purchase-timing decisions. Journal of Consumer Research 27 (4), 447–460.

Coricelli, G., Critchley, H.D., Joffily, M., O' Doherty, J.P., Sirigu, A., Dolan, R.J., 2005. Regret and its avoidance: a neuroimaging study of choice behavior. Nature Neuroscience 8 (9), 1255–1262.

Dodonova, A., Khoroshilov, Y., 2005. Applications of regret theory to asset pricing. Unpublished working paper, University of Ottawa.

Gardner, G.W., Wuilloud, T., 1995. Currency risk in international portfolios: how satisfying is optimal hedging. Journal of Portfolio Management 21, 59–67.

Gilovich, T., Medvec, V.H., 1995. The experience of regret: what, when, and why. Psychological Review 102, 379–395.

Gollier, C., Salanié, B., 2006. Individual decisions under risk, risk sharing and asset prices with regret. Unpublished working paper, Université de Toulouse, IDEI.

Gomes, F., 2005. Portfolio choice and trading volume with loss-averse investors. Journal of Business 78, 675–706.

Gul, F., 1991. A theory of disappointment aversion. Econometrica 59 (3), 667–686.

Harris, L., 2004. Is There Still Alpha to be Gained in Active Currency Management. Russell/Mellon.

Hau, H., Rey, H., 2006. Exchange rates, equity prices, and capital flows. Review of Financial Studies 19 (1), 273–297.

Kahneman, D., Lovallo, D., 1993. Timid choices and bold forecasts: a cognitive perspective on risk taking. Management Science 39, 17–31.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decisions under risk. Econometrica 47, 263–291.

Kahneman, D., Tversky, A., 1982. The psychology of preferences. Scientific American 246, 160–173.

Krähmer, D., Stone, R., 2005. Regret in Dynamic Decision Problems. Unpublished working paper, University College London.

Kroll, Y., Levy, H., Markowitz, H.M., 1984. Mean–variance versus direct utility maximization. Journal of Finance 39, 47–61.

Levy, H., Markowitz, H.M., 1979. Approximating expected utility by a function of mean and variance. American Economic Review 69, 308–317.

Loistl, O., 1976. The erroneous approximation of expected utility by means of a Taylor expansion: analytic and computational results. American Economic Review 66, 904–910.

Loomes, G., Sugden, R., 1982. Regret theory: an alternative theory of rational choice under uncertainty. Economic Journal 92, 805–824.

Muermann, A., Mitchell, O., Volkman, J., 2006. Regret, portfolio choice, and guarantees in defined contribution schemes. Insurance: Mathematics and Economics 39 (2), 219–229.

Perold, A.F., Schulman, E.C., 1988. The free lunch in currency hedging: implications for investment policy and performance standards. Financial Analysts Journal, 45–50.

Pratt, J.W., 1964. Risk aversion in the small and in the large. Econometrica 32, 122–136.

Quiggin, J., 1994. Regret theory with general choices. Journal of Risk and Uncertainty 8, 153–165.

Samuelson, P.A., 1970. The fundamental approximation theorem of portfolio analysis in terms of means, variance and higher moments. Review of Economic Studies 37, 537–542.

Savage, L.J., 1954. The Foundations of Statistics. In: Wiley Publications in Statistics. Wiley, New York.

Simonson, I., 1992. The influence of anticipating regret and responsibility on purchase decisions. Journal of Consumer Research 19, 105–118.

Smith, R.D., 1996. Is regret theory an alternative basis for estimating the value of health care interventions. Health Policy 37, 105–115.

Solnik, B., 1974. An equilibrium model of the international capital market. Journal of Economic Theory 8, 500–524.

Sorum, P.C., Mullet, E., Shim, J., Bonnin-Scaon, S., Chasseigne, G., Cogneau, J., 2004. Avoidance of anticipated regret: the ordering of prostate-specific antigen tests. Medical Decision Making 24, 149–159.

Statman, M., 2005. Hedging currencies with hindsight and regret. The Journal of Investing Summer.

Thaler, R.H., 1999. Mental accounting matters. Journal of Behavioral Decision Making 12, 183–206.

Tversky, A., Kahneman, D., 1981. The framing of decisions and the psychology of choice. Science 211, 453–458.

Zeelenberg, M., 1999. Anticipated regret, expected feedback and behavioral decision making. Journal of Behavioral Decision Making 12, 93–106.

Zeelenberg, M., van Dijk, W.W., Manstead, A.S.R., van der Pligt, J., 2000. On bad decisions and disconfirmed expectancies: the psychology of regret and disappointment. Cognition and Emotion 14, 521–541.